

ANALYSIS

# Risk Simulations of Perpetual Contracts on Digital Assets



# Risk Simulations of Perpetual Contracts on Digital Assets

Enhancing Quantitative Risk Management with the Serenity\* System

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## Abstract

This paper, forming part of a series dedicated to the computational aspects of risk management in digital assets, delves into the risk associated with perpetual contracts on digital assets. We begin by integrating perpetual contracts into a general simulation framework, highlighting the two primary variables: the underlying asset price and the basis multiplier. The study presents detailed derivations of risk simulation formulas for perpetual futures, encompassing both linear and inverse contract types. A practical case study is conducted featuring a portfolio strategy that involves a long position in Bitcoin coupled with a short position in an inverse perpetual contract on Bitcoin. The paper culminates in a comprehensive analysis of the numerical results, focusing on risk and gain metrics, and unveils an asymmetric risk profile inherent in such investment strategies.

## Contents

<b>1</b>	<b>Introduction</b>	<b>2</b>
<b>2</b>	<b>Pricing Functions</b>	<b>3</b>
2.1	Linear Perpetual Contracts . . . . .	3
2.2	Inverse Perpetual Contracts . . . . .	4
2.3	Long/Short Portfolios . . . . .	4
<b>3</b>	<b>Risk Simulations</b>	<b>5</b>
3.1	The Historical Simulation Method . . . . .	5
3.2	Risk Function for Linear Perpetuals . . . . .	6
3.3	Risk Function for Inverse Perpetuals . . . . .	6
3.4	Risk Function for a Long/Short Portfolio . . . . .	6
3.5	Consequences of Delta Hedging . . . . .	7
<b>4</b>	<b>Numerical Results</b>	<b>8</b>
4.1	The Asset Portfolio . . . . .	9
4.2	The Inverse Perpetual Portfolio . . . . .	9
4.3	The Long-Short Portfolio . . . . .	10
4.4	Gain/Risk Skew . . . . .	10
<b>5</b>	<b>Summary and Conclusions</b>	<b>11</b>

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# 1 Introduction

Perpetual contracts in digital asset markets represent a significant innovation, combining the features of traditional futures with the added advantage of lacking an expiration date. This key attribute enables traders to indefinitely speculate on the price movements of an underlying asset without actual possession, as detailed in reference [1]. The elimination of a fixed maturity date simplifies the investment process by removing the necessity for contract rollovers and enhances market liquidity through the trading of a singular contract per underlying asset. Currently prevalent in cryptocurrency markets, perpetual contracts are expected to gain traction in various other asset classes.

In a well-functioning perpetual futures market, it is expected that the contract price closely aligns with the spot price of the underlying asset. Nevertheless, market dynamics can induce temporary deviations between perpetual contract quotes and spot prices, leading to either a premium (positive basis) or a discount (negative basis). In contrast to traditional futures, which are constrained by finite maturities ensuring price convergence at expiry, perpetual futures utilize periodic funding payments. These payments, which consist of a premium reflecting the price spread and an interest component based on the interest rate differential, play a crucial role in consistently aligning the perpetual-contract price with the spot price, thus maintaining the integrity and coherence of the market.

**Linear and inverse contracts** In the dynamic realm of perpetual futures, a standard linear contract typically involves a pair like BTC/USDT, with contract sizes in the base asset (e.g., 1 BTC) and settlements in the quote currency (USDT). However, cryptocurrency trading platforms have innovated with variations to cater to diverse trading needs and regulatory constraints. A significant variation is the inverse contract, where the base currency (e.g., BTC) is used for both margin and settlement and the contract size is in the quote currency (e.g., 10,000 USD). This allows speculation on crypto-fiat exchange rates without holding fiat currency, a crucial adaptation for platforms restricted from handling fiat deposits. Another variant, which is not covered in this work, is the perpetual quanto futures, using a third currency different from the base and quote for margining and settlement. This adds complexity and new strategic opportunities to trading.

**Approaches to the perpetual-contract risk** Evaluating perpetual-contract risk can be approached from two distinct perspectives. The first is a fundamental analysis, which emphasizes the intrinsic value of the underlying asset and examines the dynamics of supply and demand in the market. This method involves modeling both market dynamics and participant behaviors to estimate the future values of the contracts.

The second method adopts a quantitative perspective. It involves closely observing market quotes for perpetual contracts and spot prices, utilizing this data to simulate future risk distributions associated with the contracts. Our focus in this paper is on the quantitative approach, wherein the contract quote and the spot price serve as the primary drivers for risk simulation analysis. In order to simplify this approach even further, we assume that the investment period to coincide with the period of premium payment for the perpetual contracts, i.e. the funding period. Therefore we perform risk simulations on a period of eight hours, which is the typical funding period for perpetual contracts.

We begin with a comprehensive examination of the pricing and risk functions specific to perpetual contracts, with an emphasis on both linear and inverse contract structures. This analysis is crucial for developing risk simulations that enable us to first calculate the risk distribution and subsequently derive the risk measures. Our primary objective is to offer a clear and detailed understanding of



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the risks associated with perpetual futures. As such, our attention will be centered on the risk simulations related to the Profit and Loss (P&L) for three distinct portfolios:

1. A portfolio with a single long position in the underlying digital asset,
2. A portfolio with a single long position in an inverse perpetual futures contract,
3. A mixed portfolio comprising a long position in the spot asset coupled with a short position in an inverse perpetual futures contract.

For each portfolio type, we will present numerical results encompassing both risk and gain measures.

## 2 Pricing Functions

In a previous paper, see reference [2], we discussed a generic framework for risk simulations of digital assets. The framework is based on the assumption that for each financial instrument there exist a pricing function for their evaluation that depends on a set of parameters and a set of pricing variables.

Therefore, it is assumed that given any financial instrument we can always write the instrument price, or value, at a future time  $t$  as:

$$F_t = f(v_1, \dots, v_n; t), \quad (1)$$

where  $f$  is the pricing function,  $v_1, \dots, v_n$ , are the pricing variables, and we have implicitly assumed that the pricing function depends on certain number of fixed parameters, which we have not made explicit. As shown in reference [2], in order to perform risk simulations, we are interested in computing the projected Profit & Losses (later simply denoted as P&L) at the end of the investment period:

$$\Delta F_t = f(v_1, \dots, v_n; t) - f(v_1^0, \dots, v_n^0; t), \quad (2)$$

where  $v_1^0, \dots, v_n^0$ , indicate the values of the pricing variables at the beginning of the investment period. Usually, the values  $v_1^0, \dots, v_n^0$ , are the latest available market quotes for the pricing variables. Note that the P&L at the end of the period, as expressed in equation (2), could be either be given by an explicit analytical formula or by a numerical evaluation.

As mentioned in section 1, we are going to focus on two different types of perpetual contracts: linear and inverse.

### 2.1 Linear Perpetual Contracts

In linear perpetual contracts, the contract size is specified in terms of the base asset and the settlement is in the quote asset. The quote asset is typically a stablecoin, such as USDT or USDC, which is loosely pegged to the value of a fiat currency. For example a linear perpetual contract for the BTC-USDT pair has a contract size of 1 BTC and is settled in USDT.

The projected P&L at the end of the investment period for a linear perpetual futures is given by:

$$\Delta F_t^L = P_t - P_0,$$

where we assumed a unitary notional and contract size,  $P_t$  is the instrument quote at the end of the investment period, and  $P_0$  is the quote at the beginning of the contract. As shown in reference [4],



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the quote of the perpetual futures at time  $t$  can always be expressed as the product of the basis multiplier  $1 + B_t$  and the spot price  $S_t$ , both observed at time  $t$ :

$$P_t = S_t \cdot (1 + B_t). \quad (3)$$

Substituting this expression in the P&L formula, we obtain:

$$\Delta F_t^L = S_t \cdot (1 + B_t) - P_0, \quad (4)$$

which can be identified as the pricing function for the P&L of a portfolio consisting of a single linear perpetual contract.

From equation (3) we notice that when the basis multiplier is positive the perpetual quote is higher than the spot price, and when the basis multiplier is negative the perpetual quote is lower.

## 2.2 Inverse Perpetual Contracts

In inverse perpetual contracts the contract size is specified in terms of the quote asset, and the settlement is in the base asset. The quote asset is typically a fiat currency, such as USD. For example an inverse perpetual contract for the BTC-USD pair could have a contract size of 10,000 USD and is settled in BTC.

The projected P&L for an inverse perpetual futures at the end of the investment period is given by:

$$\Delta F_t^I = C_s \cdot \left(1 - \frac{P_0}{P_t}\right),$$

where we assumed a unitary notional and a contract size  $C_s$ , again  $P_t$  is the perpetual quote at the end of the investment period, and  $P_0$  is the quote at which the contract is struck. Note that, unlike the majority of financial instruments, the P&L of inverse perpetuals is not linearly related to their quote.

Similarly to the linear perpetuals, here we assume that the perpetual quote can always be expressed as the product of the basis multiplier and the spot price, as shown in equation (3). Therefore, substituting this expression in the P&L formula, we obtain:

$$\Delta F_t^I = C_s \left[1 - \frac{P_0}{S_t \cdot (1 + B_t)}\right], \quad (5)$$

which can be identified as the pricing function for the P&L of a portfolio consisting of a single inverse perpetual futures.

## 2.3 Long/Short Portfolios

One of the most common strategies in perpetual trading is to use these instruments as hedging tools for *physical* positions. Consider the case in which a large amount of the digital asset, for example Bitcoin, is held in cold storage and is not easily accessible for trading. When there is market turmoils the investor may want to protect the value of the asset and may decide to go short on an inverse perpetual futures with a similar notional as the *physical* position.

We assume the investors to be long  $q$  units of the asset and that he has entered into a short-position contract in an inverse perpetual futures with a nominal value of

$$N = q \cdot S_0,$$



where  $S_0$  is the spot price of the asset in the quote currency. Note that, in order to simplify the formulas, here we assume that the contract size to be 1. In this case the portfolio P&L can be computed as

$$\Delta F_t^{\text{LS}} = q \cdot (S_t - S_0) - q \cdot S_0 \left[ 1 - \frac{P_0}{S_t \cdot (1 + B_t)} \right],$$

which can be simplified by collecting the  $q \cdot S_0$  terms as,

$$\Delta F_t^{\text{LS}} = q \cdot S_0 \left[ \frac{S_t}{S_0} - 1 + \frac{P_0}{S_t \cdot (1 + B_t)} - 1 \right]. \quad (6)$$

This expression can be considered the pricing function for the P&L of the long-short portfolio.

### 3 Risk Simulations

While all pricing functions seen so far, i.e. equations (4), (5), and (6), are markedly different, they all have the spot price and the basis multiplier as their pricing variables. As shown in reference [2], in order to perform risk simulations, we need to determine the risk drivers so that we can build the risk functions. While in the general case the pricing variables and the risk drivers may not be the same, here we assume that there is a one-to-one mapping between the pricing variables and the risk drivers.

#### 3.1 The Historical Simulation Method

We are going to use the historical simulation method to obtain the risk simulations for the P&L of the above portfolios. Hence, consider the time series of the spot price and the basis multiplier and denote them as  $S_0, \dots, S_N$ , and  $B_0, \dots, B_N$ . Here  $S_0$  is the most recent observation of the spot price and  $S_N$  is the oldest. Similarly,  $B_0$  is the most recent observation of the basis multiplier and  $B_N$  is the oldest. As described in reference [5], we assume the historical observations to be equally spaced in time, however they may not necessarily have the same frequency as the investment period. Indeed we also assume the number  $h$  to be the ratio of the investment period to the frequency of the historical observations, so that if the observations are hourly and the investment period is of one day, we have  $h = 24$ .

Since we are assuming the investment period to coincide with the funding period, in the simulation discussed in this paper we set  $h = 8$ . Given this assumption the perpetual funding is a constant that is known at the beginning of the investment period and is not subject to any uncertainty. Therefore we can include the cost of funding in the initial price of the perpetual contract, i.e.  $P_0$ .

We then define the two simulation variables:

$$s_k = \log \left( \frac{S_k}{S_{k+1}} \right), \quad (7)$$

and

$$b_k = \log \left( \frac{1 + B_k}{1 + B_{k+1}} \right), \quad (8)$$

for  $k=0, \dots, N-1$ . Note that these are the log ratios between the variables at one past period and those at the next one. As shown in reference [5] for the spot price, we can use the square-root rule on  $h$  to obtain the risk-driver simulations at the investment horizon:

$$\begin{aligned} S_k &= S_0 \cdot \exp \left( s_k \cdot \sqrt{h} \right), \\ 1 + B_k &= (1 + B_0) \cdot \exp \left( b_k \cdot \sqrt{h} \right), \end{aligned}$$



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where, again,  $S_0$  and  $B_0$  are the latest spot price and the basis multiplier.

In order to simplify the formulas that follow we assume that the perpetual contract has been struck at the beginning of the investment period, so that  $S_0$  and  $B_0$  are consistent with  $P_0$ . The general case in which the contract is struck at a different time is not much more complicated and does not alter the main results.

In the following subsections, in order to obtain the risk simulations for the portfolio P&L we substitute  $S_k$  for  $S_t$  and  $B_k$  for  $B_t$  in the P&L pricing functions, thus obtaining the risk functions in term of the simulation variables  $s_k$  and  $b_k$ .

### 3.2 Risk Function for Linear Perpetuals

The risk simulations for the P&L of the linear perpetual contracts can be obtained by using the historical simulation method on the P&L defined in equation (4) in terms of the simulation variables  $s_k$  and  $b_k$ :

$$\begin{aligned}\Delta F_k^L &= S_k \cdot (1 + B_k) - P_0 \\ &= S_0 \cdot \exp\left(s_k \cdot \sqrt{h}\right) \cdot \\ &\quad \cdot (1 + B_0) \cdot \exp\left(b_k \cdot \sqrt{h}\right) - P_0 \\ &= P_0 \left\{ \exp\left[(s_k + b_k) \cdot \sqrt{h}\right] - 1 \right\},\end{aligned}$$

for  $k = 0, \dots, N-1$ . This expression defines the risk function for the  $k$ -th scenario for cash the profit & loss  $\Delta F_k^L$  and it can be used, for example, to compute the risk simulation of the P&L of the linear perpetual contracts.

### 3.3 Risk Function for Inverse Perpetuals

Similarly we can write the risk function for the  $k$ -th scenario for the P&L of an inverse perpetual contracts by applying the historical simulation method to the pricing function defined in equation (5):

$$\begin{aligned}\Delta F_k^I &= C_s \left[ 1 - \frac{P_0}{S_k \cdot (1 + B_k)} \right] \\ &= -C_s \left\{ \exp\left[-(s_k + b_k) \cdot \sqrt{h}\right] - 1 \right\},\end{aligned}$$

again, for  $k = 0, \dots, N-1$ . This expression defines the risk function for the  $k$ -th cash scenario  $\Delta F_k^I$ , which can be used to compute the risk simulation of the inverse perpetual contracts.

Note that the expression for the P&L of the linear perpetual contracts and that of the inverse perpetual contracts, i.e. the expressions for  $\Delta F_k^L$  and  $\Delta F_k^I$ , are very similar, the main difference being the sign inside the exponential function and the substitution of the constant  $P_0$  with the constant  $-C_s$ .

### 3.4 Risk Function for a Long/Short Portfolio

Finally, we can write the risk function for the  $k$ -th scenario for the P&L of a long-short portfolio with a long position in  $q$  units of the spot asset and a short position with a notional of  $q \cdot S_0$  in the

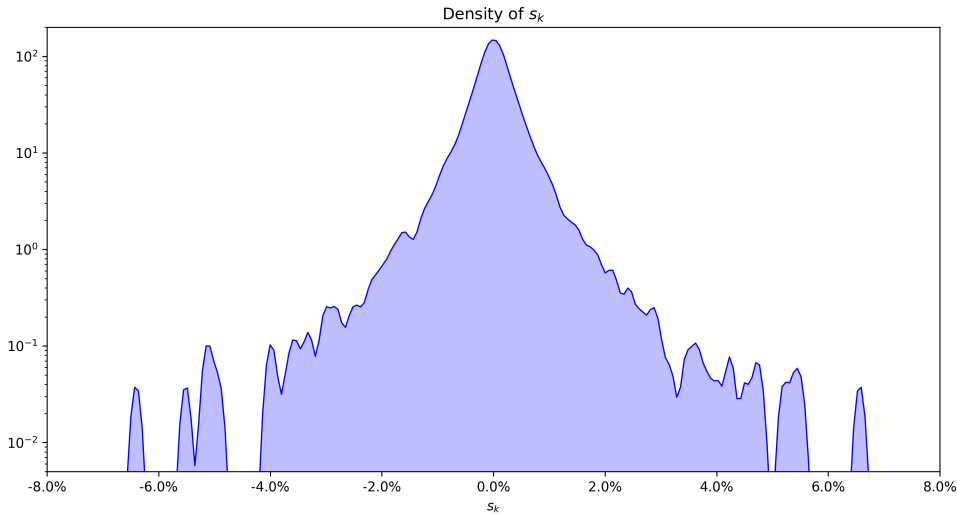


Figure 1: Density distribution of the spot simulation variable  $s_k$  as defined in equation (7).

inverse perpetual contracts. For simplicity we look at the P&L per unit of notional, i.e. we consider the P&L as written in equation (6) divided by  $q \cdot S_0$ :

$$\frac{\Delta F_t^{\text{LS}}}{q \cdot S_0} = \frac{S_t}{S_0} - 1 + \frac{P_0}{S_t \cdot (1 + B_t)} - 1.$$

By applying the historical simulation method to this function we obtain:

$$\begin{aligned} \frac{\Delta F_k^{\text{LS}}}{q \cdot S_0} &= \exp \left[ s_k \cdot \sqrt{h} \right] - 1 + \\ &\quad + \exp \left[ - (s_k + b_k) \cdot \sqrt{h} \right] - 1, \end{aligned} \quad (9)$$

again, for  $k = 0, \dots, N-1$ . This is the risk function for the P&L of the long-short portfolio.

### 3.5 Consequences of Delta Hedging

We are going to prove here that the long-short portfolio is properly delta-hedged against the spot price risk. We do this by expanding in Taylor series the expression for the long-short portfolio risk function defined in equation (9). Recall that the first-order Taylor expansion of the exponential function:

$$\exp(x) - 1 \approx x \quad \text{for small } x.$$

Hence, equation (9) for  $k = 0, \dots, N-1$  becomes:

$$\frac{\Delta F_k^{\text{LS}}}{q \cdot S_0} \approx s_k \cdot \sqrt{h} - (s_k + b_k) \cdot \sqrt{h} = b_k \cdot \sqrt{h}, \quad (10)$$

where both  $s_k$  and  $b_k$  are assumed to be small numbers. Here we notice that the simulation variable  $s_k$  is not present in this approximation of the risk function, this means that the risk of the long-short portfolio is highly sensitive to the basis multiplier volatility, and not so much to the spot price volatility.



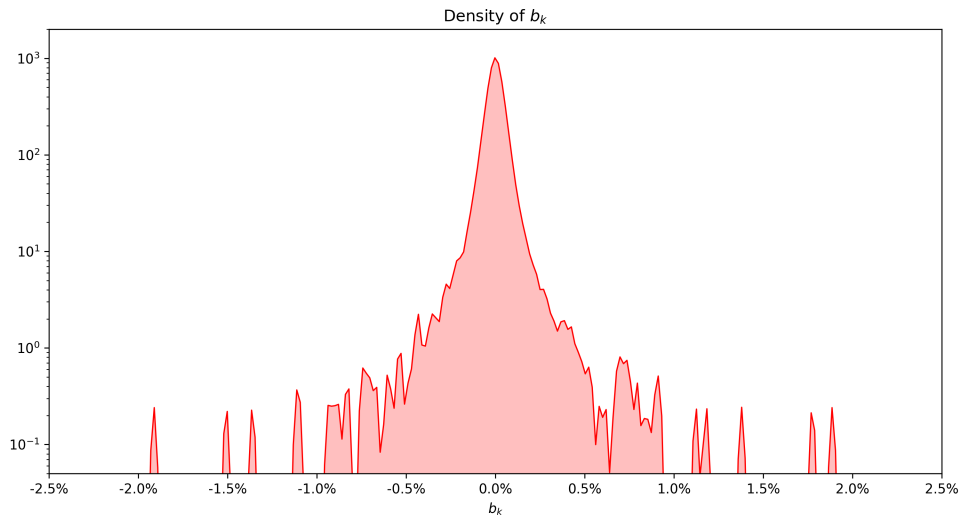


Figure 2: Density distribution for the basis simulation variable  $b_k$  as defined in equation (8).

This result can be used also in the opposite way: an investor that wants exposure to the basis multiplier needs to set up a portfolio with a long position in the digital asset (or a long position in a linear perpetual futures) and an equivalent short position in inverse perpetual futures.

## 4 Numerical Results

In this section we show some numerical results for the risk simulations of the P&L of portfolios with the base asset and an inverse perpetual contracts. We consider the case where the base asset is Bitcoin and the quote currency is USD. The historical data for both the price and the perpetual quote were observed hourly for the second half of 2022 and the whole year 2023, for a total of more than 13,200 observations. This period was chosen because it includes both periods of bear markets and bull markets. For example it includes the market turmoils due to the collapse of FTX at the end of 2022 and the recovery of the market in the last quarter of 2023.

We computed the simulation variables  $s_k$  and  $b_k$ , as defined in equations (7) and (8) for the given period. The observed density distributions for  $s_k$  and  $b_k$  are plotted in figures 1 and 2 respectively. We notice that, as expected, both distributions are centered around zero and symmetric. The distribution for the spot simulation variable  $s_k$  is wider than the distribution for the basis simulation variable  $b_k$  and we computed an annualized volatility of 47.0% for the former and 7.7% for the latter. The correlation between  $s_k$  and  $b_k$  is very small and negative, around -0.06. This stylized fact is consistent with the results that the basis multiplier can be considered a separate risk driver from the spot price.

We then proceeded to compute the risk simulations for the P&L of the spot, the perpetual, and the long-short portfolios.



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	VaR 99%	VaR 95%	CVaR 95%	CVaR 99%
Asset	42,625	19,085	33,532	62,464
Inverse perpetual	44,390	19,235	35,121	67,972
Long/short portfolio	4,889	2,144	4,585	10,836

Table 1: Risk measures for the P&L of the spot, the perpetual, and the long-short portfolio. The risk measures are expressed in USD for a notional amount of 1,000,000 USD.

	GaR 99%	GaR 95%	CGaR 95%	CGaR 99%
Asset	43,598	19,670	35,442	67,502
Inverse perpetual	41,370	19,204	33,922	63,184
Long/short portfolio	7,478	2,791	6,140	14,088

Table 2: Gain measures for the P&L of the spot, the perpetual, and the long-short portfolio. The gain measures are expressed in USD for a notional amount of 1,000,000 USD.

## 4.1 The Asset Portfolio

First we consider a simple portfolio consisting of a long position in the spot asset. The P&L pricing function for this portfolio is given by:

$$\Delta F_t^S = N \cdot \left( \frac{S_t}{S_0} - 1 \right),$$

where  $N$  is the notional amount of the portfolio, which in the numerical computations is set to 1,000,000 USD, and  $S_t$  is the spot price at time  $t$ . Note the absence of the basis multiplier in this expression, which is consistent with the fact that the portfolio is not exposed to the basis risk.

Using the historical simulation method we can compute the risk function for the P&L of the spot portfolio as

$$\Delta F_k^S = N \cdot \left( \exp \left[ s_k \cdot \sqrt{h} \right] - 1 \right).$$

In the numerical simulations described above we computed the risk measures, i.e. the Value-at-Risk (VaR) and the Conditional Value-at-Risk (CVaR), for both the 95% and 99% confidence levels. We also computed the gain measures, i.e. the Gain-at-Risk (GaR) and the Conditional Gain-at-Risk (CGaR), for the same confidence levels. More details on the definitions of these risk and gain measures can be found in reference [5].

The results of the numerical simulations are shown in tables 1 and 2 where the spot portfolio is denoted as *Asset*. We note that in all cases the gain measures are larger than the risk measures, indicating that the gain is larger than the risk for the spot portfolio.

## 4.2 The Inverse Perpetual Portfolio

We also performed the P&L simulations for the inverse perpetual portfolio identified by the pricing function in equation (5). These simulations were also performed using a notional amount of 1,000,000 USD.

The results for the risk measures and the gain measures are also shown in tables 1 and 2. We note that in all cases the values for the risk measures are larger than the corresponding values for the gain measures, indicating that the risk is larger than the gain for the inverse perpetual portfolio. We



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	95% GaR/VaR	99% GaR/VaR	95% CGaR/CVaR	99% CGaR/CVaR
Asset	3.1%	2.3%	5.7%	8.1%
Inverse perpetual	-0.2%	-6.8%	-3.4%	-7.0%
Long/short portfolio	30.2%	52.9%	33.9%	30.0%

Table 3: Skew results for the P&L of the spot, the perpetual, and the long-short portfolio.

also notice that the risk measures for the inverse perpetual portfolio are always larger than the risk measures for the spot portfolio, indicating that the risk is larger for the inverse perpetual portfolio than for the spot portfolio. Conversely the gain measures for the inverse perpetual portfolio are always smaller than the gain measures for the spot portfolio, indicating that the gain is smaller for the inverse perpetual portfolio than for the spot portfolio.

### 4.3 The Long-Short Portfolio

Finally we performed the P&L simulations for the long-short portfolio identified by the pricing function in equation (6). As discussed in subsection 3.5, because the effects of the delta hedging, the dispersion of the P&L of the long-short portfolio is expected to be markedly smaller than that of both the spot and the inverse perpetual portfolios. The numerical results for the risk and gain measures are also shown in tables 1 and 2. We note that indeed both the risk measures and the gain measures are smaller than the corresponding measures for the spot and the inverse perpetual portfolios. Furthermore we notice that the gain measures are much larger than the corresponding risk measures,

### 4.4 Gain/Risk Skew

In the previous sections we observed an asymmetry between the risk measures and the gain measures of the investment strategy involving inverse perpetual contracts. We can quantify this asymmetry by defining the gain/risk skew. Given a risk measure  $R$  and the corresponding gain measure  $G$ , we define the gain/risk skew as:

$$\text{Skew} = \frac{G}{R} - 1, \quad (11)$$

which is a measure of the asymmetry of the gain/risk profile of the investment strategy. The skew is positive when the gain is larger than the risk and negative when the risk is larger than the gain. If the skew is close to zero, the risk and the gain are similar in magnitude. Obviously an investor would prefer to invest in a strategy with a positive skew.

In table 3 we show the results for the gain/risk skew for the P&L of the spot, the perpetual, and the long-short portfolio. We notice that the skew is consistently positive, around a few percent, for the asset portfolio. On the other hand, the skew is small and negative for the inverse perpetual portfolio. Finally, the skew is large and positive, larger than thirty percent, for the long-short portfolio. According to these results the long-short portfolio has a well-pronounced skew of the gain/risk profile, so that an investor might find attractive to invest in this portfolio.

We stress again that these are single-period results and are aimed to estimate the risk and gains in a period of eight hours, which is the typical funding period for perpetual contracts. In order to understand the long-term features of the investment strategies one would need to perform a multi-period analysis, which is beyond the scope of this paper.



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These results might be dependent on the historical data used for the simulations, however we obtained similar results when we used different historical periods. More research is needed to confirm these results and to understand the reasons behind the skew.

## 5 Summary and Conclusions

This paper provides a comprehensive exploration of pricing functions and risk simulations for various types of perpetual contracts and portfolios. We extended the framework discussed in reference [2] to include specific analyses of linear and inverse perpetual contracts, as well as long/short portfolios.

Our findings demonstrate that the pricing functions for linear and inverse perpetual contracts, see equations (4) and (5), effectively capture the dynamics of these financial instruments. The historical simulation method, as applied to these functions, provided valuable insights into the risk profiles of these contracts. Notably, our simulations revealed distinct risk characteristics for linear and inverse perpetuals, with the latter exhibiting a more complex risk profile.

The long/short portfolio introduced in section 2, as described by the pricing function of equation (6), is the most simple portfolio that one can create for a complex strategy that includes assets and perpetual futures. Our analysis indicates that such portfolios can effectively mitigate spot price volatility, and show that the portfolio main risk driver is the basis-multiplier volatility. This finding is particularly relevant for investors who wish to manage their exposure to spot-price risk.

The numerical results described in section 4, underscore the practical applications of our theoretical framework. The historical data analysis provided real-world context to our theoretical constructs, and the risk/gain measures offer valuable benchmarks for investors. In that analysis, we observed that the long/short portfolio has a well-pronounced skew of the gain/risk profile, which is positive and large, indicating that the gain is larger than the risk for this portfolio.

In conclusion, this paper advances the understanding of perpetual contracts and their applications in digital asset markets. Our analytical framework and numerical results contribute to more informed decision-making processes for investors and risk managers. Future research might explore the implications of these findings in different market conditions and for other types of digital assets.

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