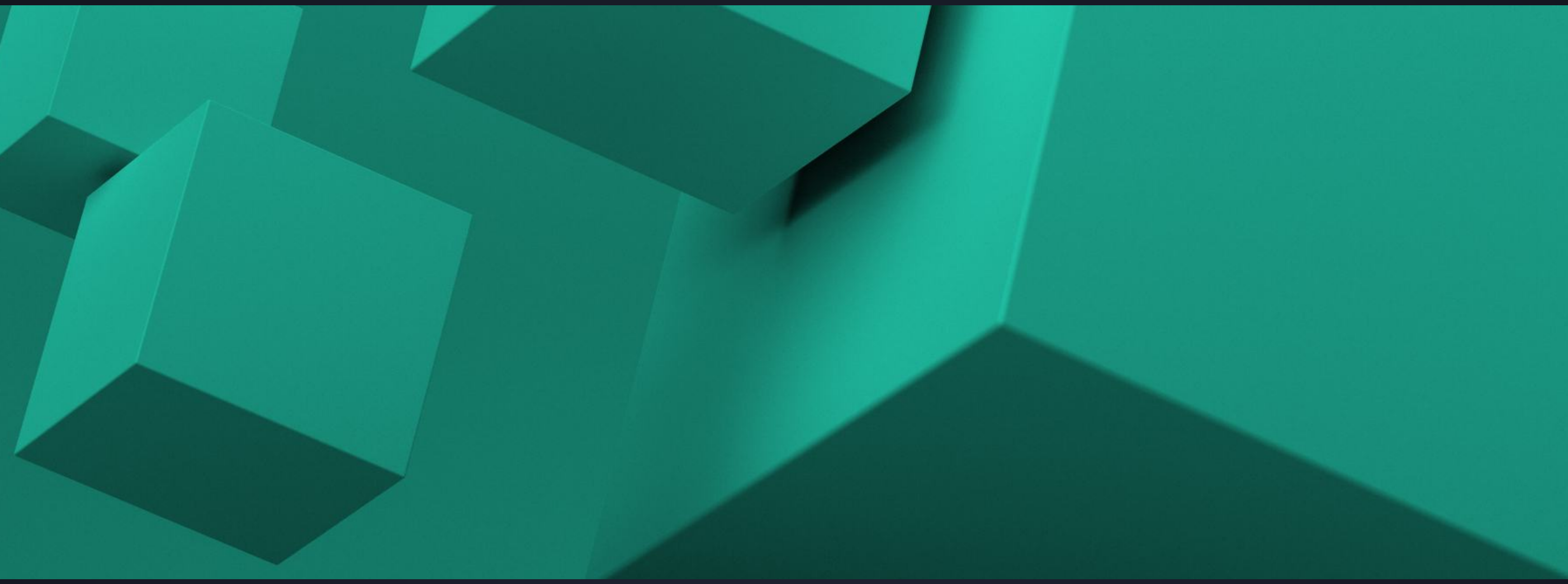
ANALYSIS

Risk Neutral Discounting for Crypto Options



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Risk Neutral Discounting for Crypto Options in Serenity*

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1 Abstract

We review various approaches to risk neutral discounting for options on crypto currencies. In particular, we examine sources of crypto lending rates as well as benchmark indices constructed to represent a risk free rate of return for crypto markets. We compare these rates with the discount rates implied from future and option prices on Deribit. We conclude with a discussion of the approach to discounting used in Serenity.

2 Introduction

Serenity provides pricing and Greek analytics for European options traded on Deribit[1]. In order to price these options in the context of risk-neutral evaluation, we have to choose which discount curve to use. In traditional finance, one constructs a discount curve from liquidly-traded instruments whose yield represents, in some sense, a risk neutral rate of return. We seek an analogous approach from crypto derivatives.

Since the end of the financial crisis of 2008, most OTC derivatives trading has moved to "central clearing" through exchanges. These trades are subject to CSA agreements that require the posting of collateral to offset any potential losses from a counterparty credit event. In the limit of perfect collateral, one can prove [2][3] that the correct discount rate is equal to the rate of return on the posted collateral. For this reason, central cleared derivatives are priced using the discount curve built from Overnight Index Swaps, as this is effectively the rate of return on posted collateral. Unfortunately, no CSA-style margin mechanism currently exists in the crypto economy. To make matters worse, there is also no standardized *uncollateralized* interest rate benchmark, analogous to LIBOR. There are, however, some recent attempts to construct such a standardized curve for the crypto markets, which we discuss in later sections. We begin by examining DeFi lending protocols.

3 Defi Lending Protocols

DeFi lending protocols offer a novel approach to lending cryptocurrencies. The three most prominent such protocols are Aave[4], Compound[5] and Maker[6]. These protocols enable the borrowing and lending of cryptocurrencies through decentralized exchanges (DEX) where prices are determined algorithmically as a function of supply and demand. As depicted in Figure 1, borrowers take out loans from a pool of token reserves by first posting collateral in another token. The loans have no expiry (infinite duration) and can be repaid at any time. They are often *over-collateralized* in the sense that $> \%100$ of the value of the loan must be posted in the collateral currency.

Currently, exchanges such as Aave offer borrowing and lending rates on a handful of prominent tokens. For example, the lending rates available on Aave include the tokens given in Figure 2 An example of the Aave's lend and borrow rates for WBTC are given in Figure 3. An example of

*Cloudwall and the technology behind its Serenity System were acquired by Talos in April 2024.

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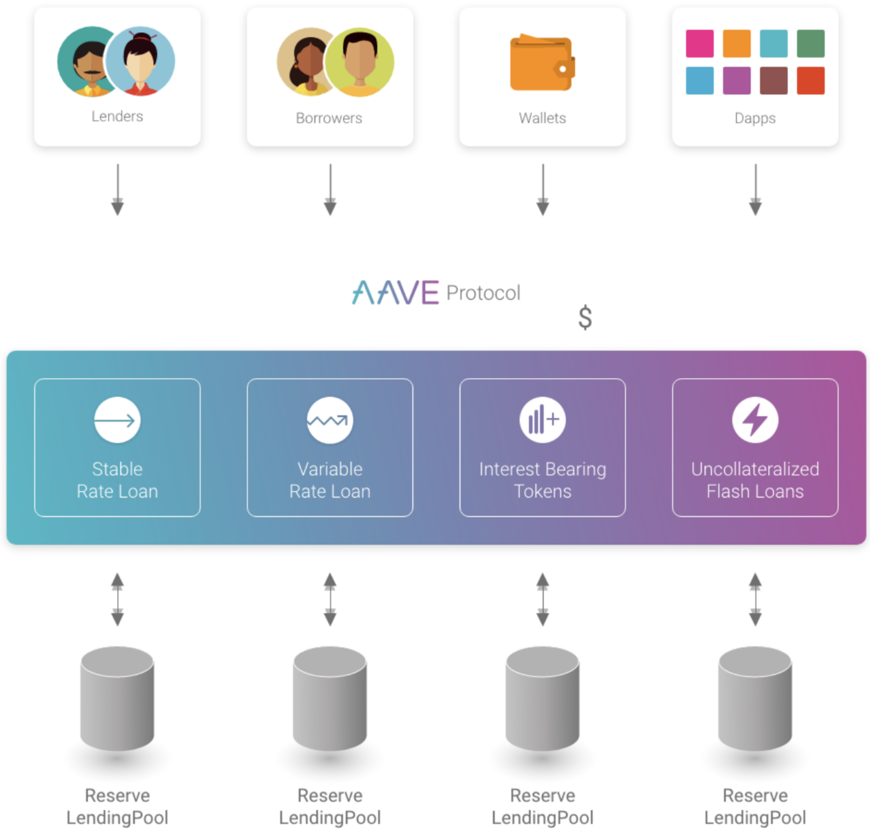


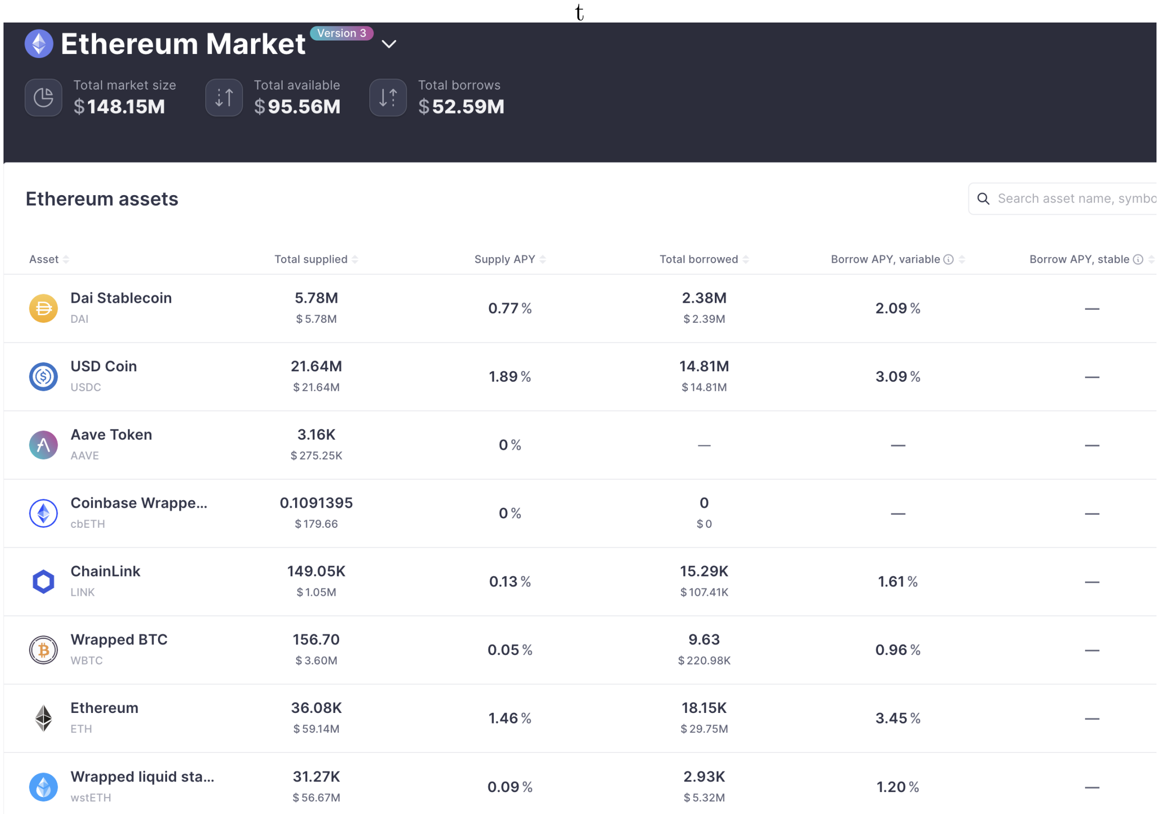
Figure 1: Aave Lending Protocol Schematic

the so-called "interest rate model" is given by the purple graph in Figure 3. The borrow rate is determined algorithmically as a function of the pool "utilization rate". As the utilization rate goes up, borrow rates also increase to limit borrowing demand and incentivize borrowing supply. As we will see later, these rates can vary quite a lot day on day.

4 IPOR

In the context of crypto-native interest rate benchmarks, IPOR[7] is an important project. IPOR stands for the Inter-Protocol Over-Block Rate. It attempts to provide a LIBOR style interest rate benchmark curve for cryptocurrencies. In particular, the IPOR protocol constructs a short rate index by averaging the lending and borrowing rates available through the DeFi lending protocols discussed above

It goes one step further, and offers vanilla interest rate swaps based on this index and traded on its own decentralized exchange. As with other decentralized exchanges, prices are set algorithmically t



Ethereum Market Version 3

Total market size $148.15M | Total available $95.56M | Total borrows $52.59M

Ethereum assets

Asset	Total supplied	Supply APY	Total borrowed	Borrow APY, variable	Borrow APY, stable
Dai Stablecoin DAI	5.78M $ 5.78M	0.77 %	2.38M $ 2.39M	2.09 %	—
USD Coin USDC	21.64M $ 21.64M	1.89 %	14.81M $ 14.81M	3.09 %	—
Aave Token AAVE	3.16K $ 275.25K	0 %	—	—	—
Coinbase Wrappe... cbETH	0.1091395 $ 179.66	0 %	0 $ 0	—	—
ChainLink LINK	149.05K $ 1.05M	0.13 %	15.29K $ 107.41K	1.61 %	—
Wrapped BTC WBTC	156.70 $ 3.60M	0.05 %	9.63 $ 220.98K	0.96 %	—
Ethereum ETH	36.08K $ 59.14M	1.46 %	18.15K $ 29.75M	3.45 %	—
Wrapped liquid sta... wstETH	31.27K $ 56.67M	0.09 %	2.93K $ 5.32M	1.20 %	—

Figure 2: Tokens Available on Aave

through pools of reserves and price oracles. A diagram of this type of swap is given in Figure 4. An example of IPOR swap rates are given in Figure 5. These rates tend to be more stable than the short rate directly available through Defi protocols. They have a term structure of 1H, 1D, 1M, 1W, 1Y but rates are only quoted in stable coins USDC, USDT and Dai.

5 BIRC

Recently, CF Benchmarks have partnered with Chainlink[8] to construct an interest benchmark curve specifically for Bitcoin referred to as the Bitcoin Interest Rate Curve BIRC[9]. This curve is bootstrapped from averages of relevant price quotes throughout the crypto economy. In particular, the various tenors of the curves are built from prices coming from 3 sources:

- DeFi lending and borrowing rates
- Implied discount rates derived from Future prices on centralized crypto exchanges
- OTC cryptocurrency lending rates

The specific sources, per tenor, are given in the Figure 6.

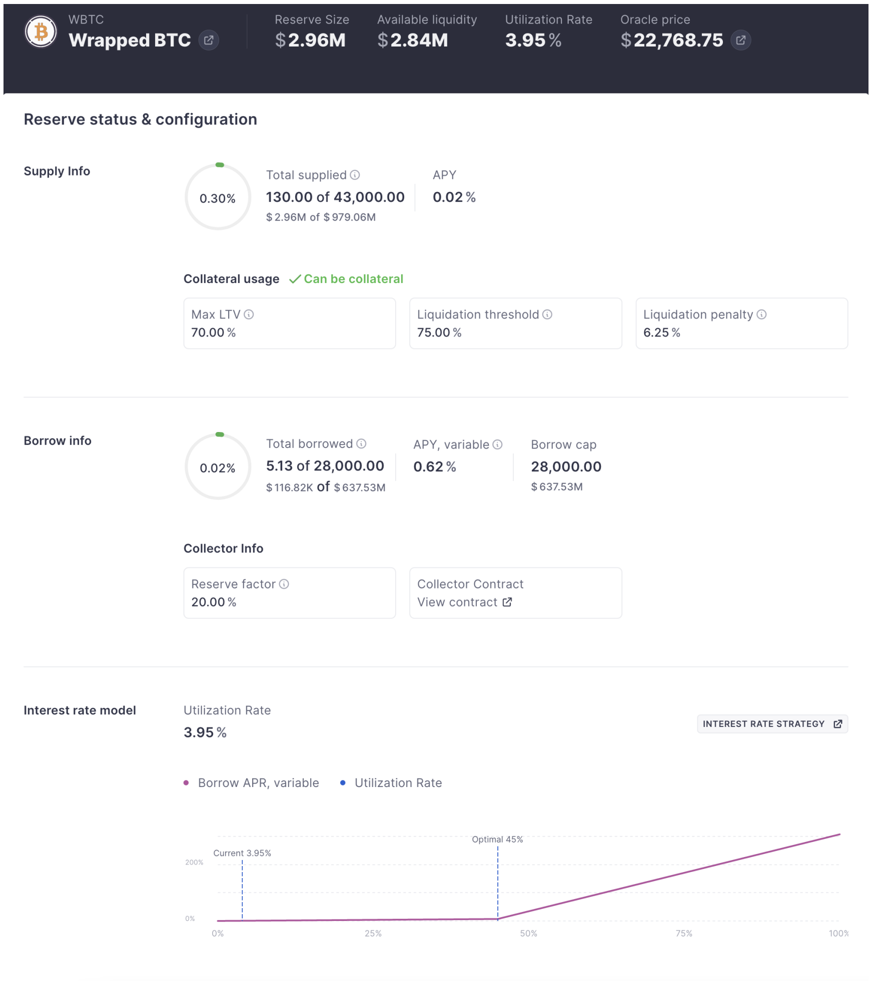


Figure 3: Example Lend and Borrow Rats for WBTC on Aave

We mentioned DeFi lending protocols earlier. We now briefly discuss how Futures prices on CeFi exchanges can provide meaningful input to an implied discount rate. Specifically, we examine how discount rates can be derived from Futures prices using traditional risk neutral parity relationships.

6 Spot-Future Parity

In the context of traditional options pricing theory, there exists a risk neutral relationship between spot and future prices called Spot-Future Parity[10]. Specifically, one can build a risk neutral portfolio with the following three positions:

- a long forward, struck at F
- a short spot, $-S$
- cash-like position equal to the value of the spot, accruing a risk free interest, r

Interest Rate Swap

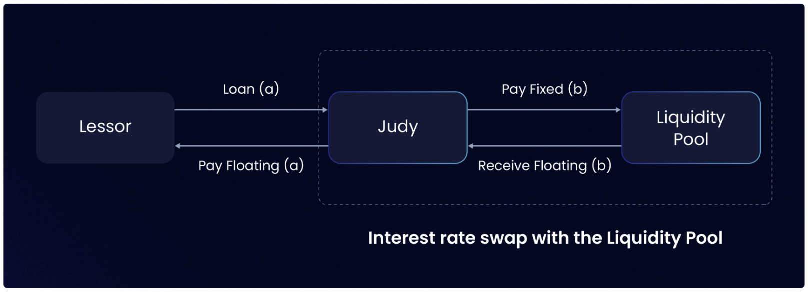


Figure 4: Interest Rate Swap cashflow diagram on IPOR

Today's value	Value at maturity
0	$S_T - F$
$-S_0$	$-S_T$
S_0	$S_T * e^{rT}$
0	$S_0 * e^{rT} - F$

Table 1: Spot Future Parity Cashflows

In table 1 we compute the value of this portfolio today and at maturity, t Since this portfolio is risk free, meaning its total value is independent of the evolution of the spot price, then its future and present value must be equivalent. This gives the risk neutral relationship between spot, S_0, and future prices F.

$$S_0 * e^{rT} = F \tag{1}$$

BIRC uses this relationship to back out the implied risk free rate from future and spot quotes on the exchanges given in Figure 7. See the documentation in BIRC [9] for more details on the specific formulas used to exact the discount factor.

Finally, BIRC also aggregates lending rates for OTC crypto lenders via a daily survey. The rates are averaged with Future implied rates to generate the BIRC index for short and medium tenors. A time series of the BIRC index is given in Figure 8:

We note that following important observations about this index:

- The index highly volatile. The day-on-day changes for all tenors can be very high. Rates as high as 60% can be observed during the recent FTX crash in November of 2022. This kind of volatility is consistent with what Serenity produces using Future prices on Derebit. It seems to be endemic to the market.

- The index can often display backwardation, with short term rates above long term rates for many dates in the time series. This is also consistent with what Serenity observes using Derebit prices.

- The rates seemed to be floored to zero which is inconsistent with the rates observed on Deribit. This may be a representation choice for BIRC.

Figure 5: 1M IPOR rate for USDC

Category	Relevant Tenors	Instruments
Session Interest Rate	Intraday	Perpetual Futures, DeFi Lending Protocols, OTC Lending
Short Term	1 - 3 Week	Fixed Maturity Futures, OTC Lending
	1 - 5 Month	
Medium Term	6 - 12 Month	Fixed Maturity Futures, OTC Lending

Figure 6: BIRC input market quotes

7 Risk Neutral Discounting for Crypto Options in Serenity

Finally, we discuss the discounting curve construction logic in Serenity. As a reminder, Serenity provides risk neutral pricing and Greek analytics for options traded on Derebit. In order generate correct analytics we need a discount curve that is consistent with prices quoted on the exchange. In the absence of a market standard discount curve for crypto currencies, we take a practical approach and derive an implied discount rate consistent with Future prices on Deribit. This is analogous to the approach taken for BIRC but with Deribit futures quotes used as input. We refer to this approach as "self discounting" in Serenity.

A plot of this implied rate for BTC on Dec 12, 2022 is given in Figure 9. As a reminder, the annualized rate of return is defined in the following formula

$$S_0 * e^{rT} = F \tag{2}$$

As mentioned above, these discount rates are the rates consistent with the futures prices and the absence of arbitrage. One might ask whether assuming no arbitrage in a nascent exchange, such as Deribit, is a sound assumption. To address that concern we examine some other no arbitrage relationships using the same data from Deribit.

8 Put-Call Parity

As is well known [12], there exists a risk neutral relationship between puts, calls and futures called Put-Call Parity[11]. Consider a put and call with the same underlying, strike and maturity. These are referred to as pairs. A portfolio that is long the call and short the put has the following payof

$$\max(S_t - K, 0) - \max(K - S_t, 0) = S_t - K \tag{3}$$

7.1 Perpetual Futures Contributing Exchanges

Contributing Exchanges	Relevant Tickers	Funding Rate Settlement Interval	CT	CAF_B	CAF_C
Binance Futures	BTCUSDT	00:00 UTC; 08:00 UTC and 16:00 UTC	24hr	-0.02%	0.02%
Bitmex	XBTUSDT	04:00 UTC, 12:00 UTC, and 20:00 UTC	24hr	-0.02%	0.02%
OKX	BTC-USDT-SWAP	00:00 UTC; 08:00 UTC and 16:00 UTC	24hr	-0.02%	0.02%
Bybit	BTCUSDT	00:00 UTC; 08:00 UTC and 16:00 UTC	24hr	-0.02%	0.02%

7.2 Fixed Maturity Futures Contributing Exchanges

Contributing Exchanges	Relevant Tickers
OKX	BTC-USDT-{expirydate}
Binance Futures	BTCUSDT_{expirydate}
Bitmex	XBTUSDT{expirysymbol}
CME	BTC{expirysymbol}{expiryyear}

Figure 7: Centralized Exchanges used in BIRC

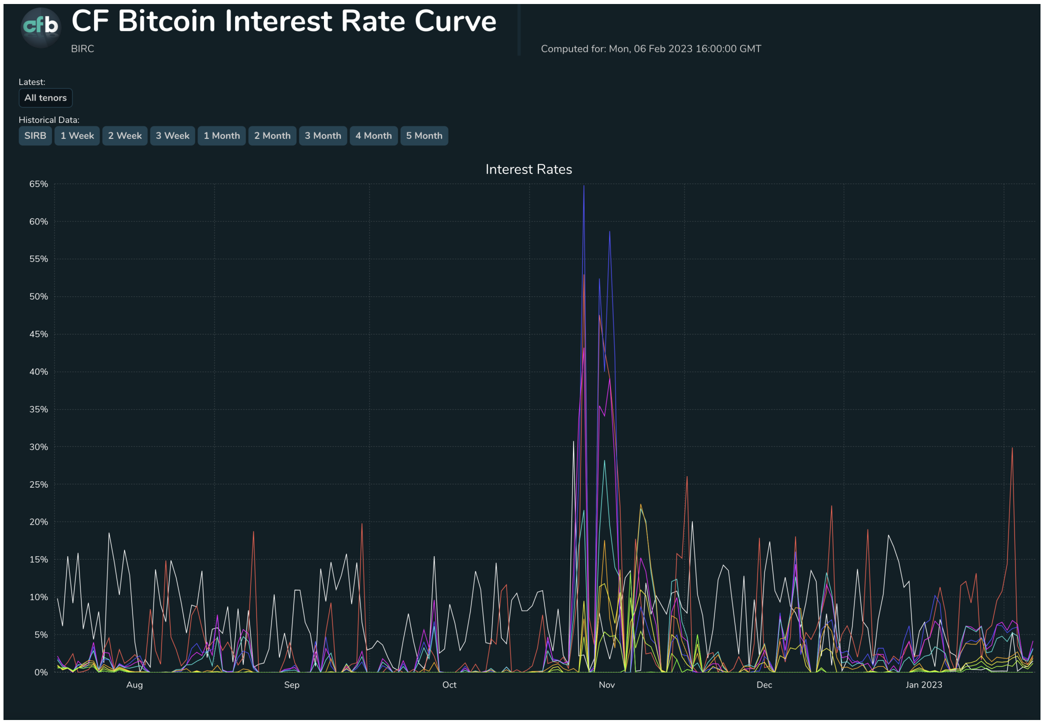


Figure 8: BIRC Index time series

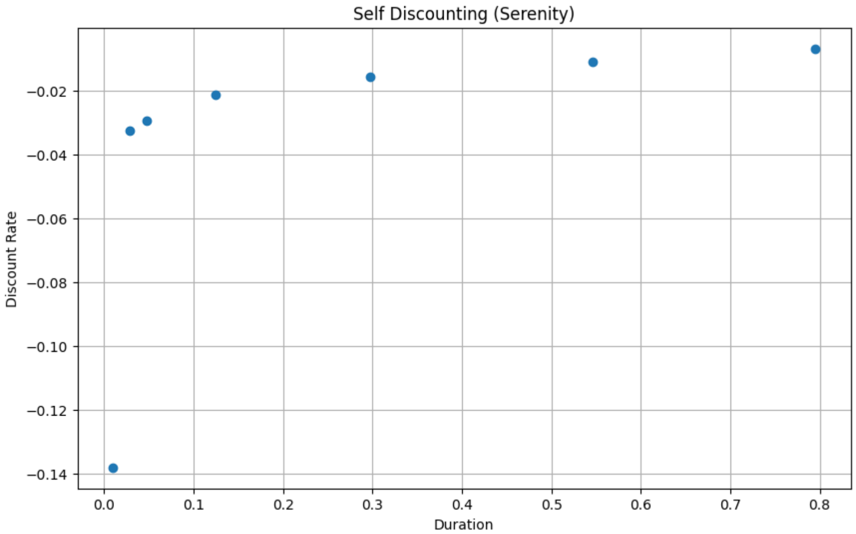


Figure 9: Discount Rates implied for spot-forward parity on Deribit, BTC Dec 12, 2022

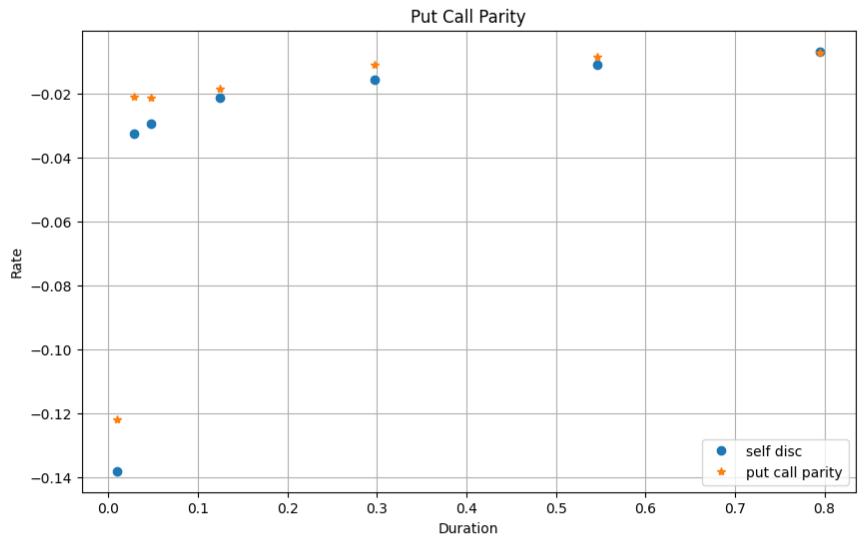


Figure 10: Discount Rates implied for put-call parity on Deribit, BTC Dec 12, 2022

Since the portfolio's value is independent of the underlying, and hence risk free, its present value must be equal to the discounted value at expiry. This leads to the following, model independent relationship

$$C - P = e^{-rt}(F - K) \tag{4}$$

Using call and put quotes, for BTC with strike \$18, 000 gives the discount rate values in Figure 10.

9 Box Trade

Finally we consider a less commonly, no-arbitrage relationship, namely a box trade. This portfolio consists of two sets of put/call pair, with each pair at a different strike(K_1, K_2) but with the same maturity. The payoff for this portfolio looks like the following

$$\max(S_t - K_1, 0) - \max(K_1 - S_t, 0) - \max(S_t - K_2, 0) + \max(K_2 - S_t, 0) = K_2 - K_1 \tag{5}$$

This payoff is a constant value irrespective of the fluctuating value of the underlying. It looks like Figure 11.

We can see that this portfolio must equal the discount difference in strike e $-e^{rt} * (K_2 - K_1)$. As before we consider options on BTC, with strike pairs of \$16000 and \$18000. We plot this relationship, in terms of the discount rate, in Figure 12.

From this analysis we conclude that the assumption of no arbitrage is sound for options traded on Deribit. All of the prices examined are internally self consistent with no arbitrage arguments. We therefore conclude that the self discounting approach, i.e. backing out implied discount rates from Future's prices is a reasonable approach to take for Serenity analytics.

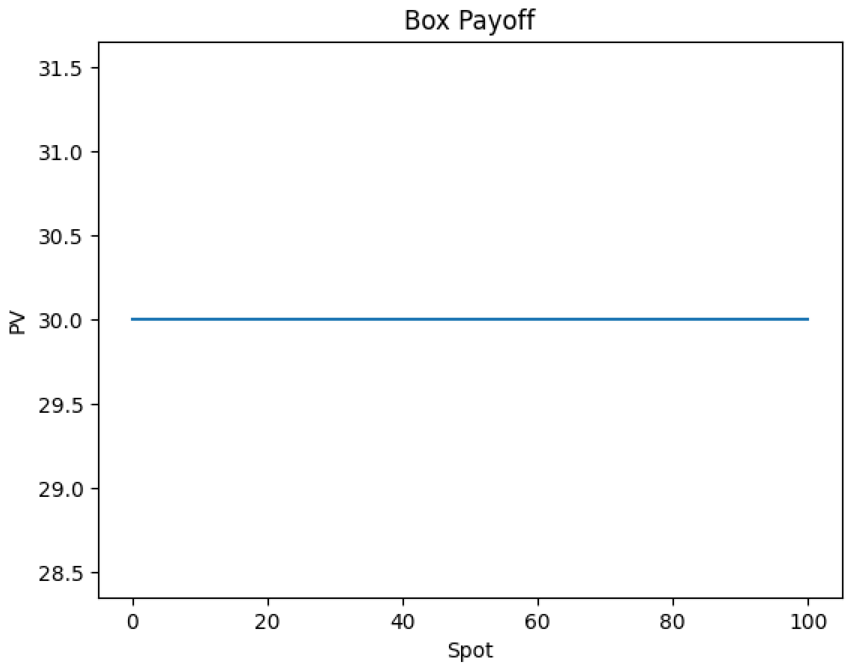


Figure 11: Example box trade payoff diagram, $K_2 - K_1 = 30$

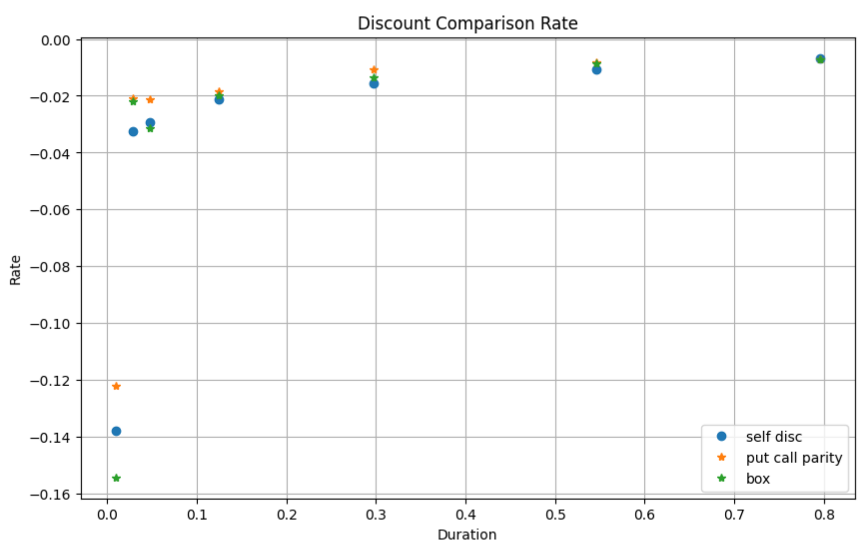


Figure 12: Implied Discount Rates from box trade, Debirit, BTC Dec 12, 2022

10 Conclusions

As a reminder, the specific, rather narrow, goal for Serenity was to select a discount curve construction logic that allows us to generate risk neutral option prices consistent with market quotes on Deribit. From the analysis above we can draw a few conclusions:

- *No Market Standard Discount Curve currently exists* All of the external sources examined above have deficiencies when it comes to discount options on Deribit.
  - **DeFi Protocols**: No obvious term structure, large variability day-on-day, no sense that these protocols represent the majority of the crypto lending present in the market
  - **IPOR**: No coverage for ETH, BTC. Underlying rates are sourced for DeFi protocols so limitations of DeFi are inherited.
  - **BIRC**: Bitcoin specific, Deribit prices not used in the index. Floored to zero.
- **For pricing on Deribit, implied discount rates are sensible** Given the limitations of the source discussed, in Serenity we imply discount rates from Futures prices. We have shown evidence for the soundness of the assumption of no arbitrage by examining other no arbitrage price constraints on Deribit price data.

We wanted to share our early results and receive feedback from the community. What would you like to understand better? Any questions this raised you'd like us to answer? We appreciate your comments.

11 Authors

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DISCLAIMER

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Cloudwall and the technology behind its Serenity System were acquired by Talos in April 2024.

talos.com