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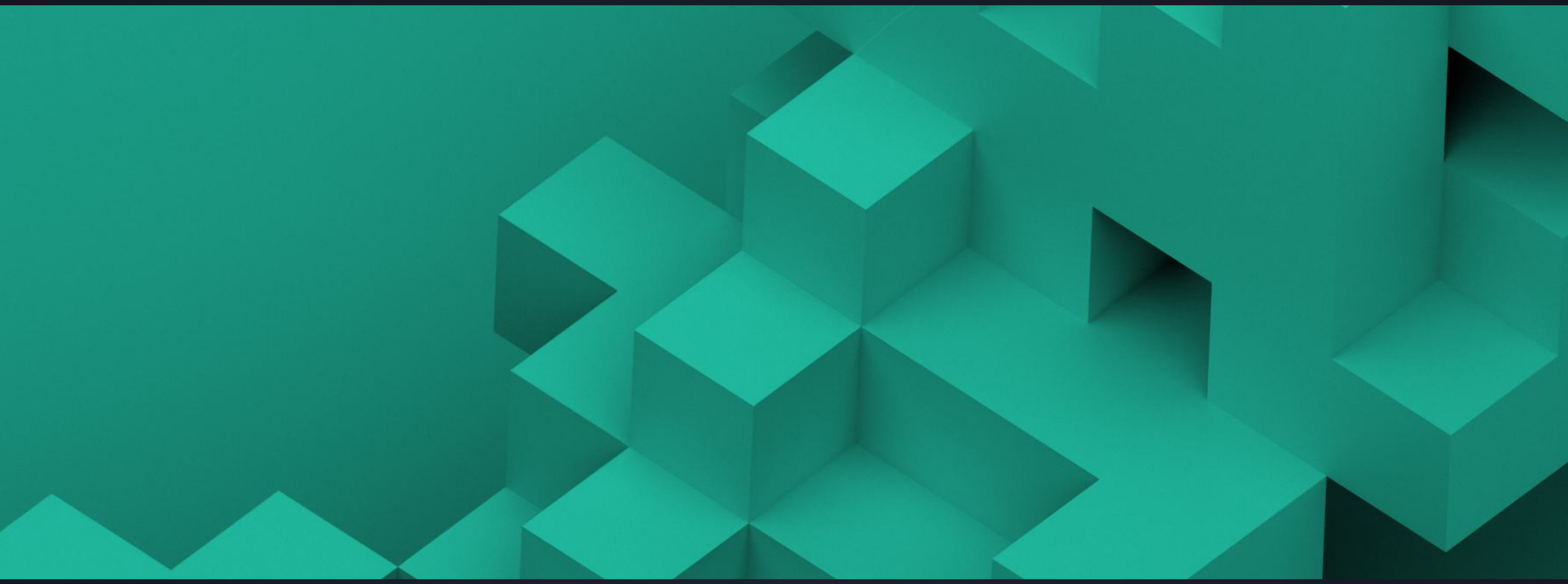
Projection Rates and Basis for Perpetuals and Futures



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Projection Rates and Basis for Perpetuals and Futures

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Abstract

Within the Cloudwall Serenity* software framework, we examine linear derivatives on digital assets, like perpetual and futures contracts. We explore various term structures to approximate market expectations of forward prices. Additionally, we introduce several basis numbers, in both multiplicative and logarithmic forms, demonstrating their application alongside the discussed term structures. It is shown how the knowledge of basis numbers and projection rates helps traders and portfolio managers to properly fine-tune their portfolio risk profile, offering insights into strategic decision-making in the dynamic world of digital asset trading.

Contents

1 Introduction

The digital asset market has grown notably with the introduction of various financial tools such as perpetual and futures contracts. These tools present traders with a range of strategies to interact with the market, each having its own unique features and effects. Perpetual contracts, which do not have an expiry date, allow traders to hold positions as long as they have the necessary collateral.

* *Cloudwall and the technology behind its Serenity System were acquired by Talos in April 2024.*

On the other side futures contracts, with set expiry dates, enable traders to take positions on asset prices at precise future times. It is crucial to understand these tools for effective risk management and strategy improvement in digital asset trading. This paper discusses the fundamental aspects of perpetual and futures contracts, explores the calculation of different types of basis and introduces projection rates useful to estimate the forward prices of digital assets.

In this dynamic environment the Serenity software by Cloudwall becomes an indispensable tool, equipping traders and investors with the knowledge and analytical capabilities to effectively manage and capitalize on the intricacies of linear derivatives on digital assets.

1.1 Market Data

At Cloudwall we believe that market data should be the center of any modern software, therefore we built Serenity as a data-driven system.

Digital asset exchanges offer a variety of financial instruments for trading asset pairs like BTC/USD. These include:

- Spot markets, facilitating the immediate purchase of a digital assets.
- A range of futures contracts, allowing traders to take positions at different maturities.
- Perpetual contracts, enabling leveraged positions not tied to any specific maturity.

These instruments provide traders with the flexibility to align their trading strategies according to their market outlook, risk tolerance, and investment horizon. The diversity in instrument types and maturity dates also enriches the market ecosystem, fostering a dynamic and comprehensive trading environment.

Note that not all exchanges offer all these asset types. For example, there are exchanges that only offer futures and perpetual contracts and do not have spot facilities. On the other hand some exchanges, such as DeFi platforms, do not significantly cover margin trading contracts. As we shall see below, this is important because trading the same asset on different exchanges may offer some arbitrage opportunities. The below definitions of basis are somewhat measuring these arbitrage opportunities.

Understanding the variety and characteristics of these instruments requires precise market data. At Cloudwall, we collect data from a multitude of providers to create data series for the Serenity software. Accurate market data is crucial for reliable analyses and decision-making in financial undertakings. To ensure the integrity and accuracy of the data, Cloudwall has implemented robust data checks, as a consequence the data utilized is of high quality. The data in table 1 illustrates the price and volume dynamics of different contract types, which is essential for traders and analysts to gauge market behavior and identify potential trading opportunities.

2 Perpetual Contracts

Perpetual contracts represent a significant innovation in the domain of financial derivatives, especially within digital asset markets. Unlike traditional futures contracts which have a predetermined expiration date perpetual contracts do not possess an expiry date, allowing positions to be held indefinitely as long as the necessary collateral is maintained.

Name	Last Price (USD)	24H Change	24H Volume (USD)
BTC-Perpetual	27,614.50	-0.99%	$323.08M
BTC-13-OCT-23	27,600.00	-1.02%	$4.13M
BTC-20-OCT-23	27,627.50	-0.99%	$2.55M
BTC-27-OCT-23	27,635.00	-1.07%	$4.47M
BTC-29-DEC-23	27,892.50	-1.09%	$7.93M
BTC-24-NOV-23	27,730.00	-1.13%	$1.54M
BTC-29-MAR-24	28,225.00	-1.14%	$3.18M
BTC-27-SEP-24	29,017.00	-0.88%	$1.64M

Table 1: Deribit market quotes for the Bitcoin/USD pair on 10 October 2023 at 6:00 UTC. Recall that futures maturities are at 8:00 UTC. Note that the BTC-USD Spot Price at the same time was observed to be 27,615.00.

The trading dynamics of perpetual contracts also differ from those of futures contracts, especially regarding the settlement process, which occurs continuously in the case of perpetual contracts as opposed to at a set date in the case of traditional futures contracts.

2.1 The perpetual contracts basis

Given a digital assets its spot price represents the current market price at which the asset can be bought for immediate delivery (or sold depending if it is a bid or an ask price). The quoted price reflects the immediate equilibrium between supply and demand for that asset. In digital-asset exchanges the spot price serves as the most straightforward representation of an asset's current value and is often considered a reference price for the associated derivatives.

On the other hand a perpetual-contract quote refers to the price at which traders can enter a contract to buy or sell a digital asset at a later time, without a predetermined expiration date. Perpetual contract quotes can deviate from the current market price of the underlying asset due to factors such as funding rates, leverage, and market sentiment. The perpetual contract quote is affected by traders' expectations of the asset's future price movements and the dynamics of the perpetual market itself.

Perpetual contracts are frequently traded at a premium or discount to the spot price, contingent on the market conditions. The basis of a perpetual contract measures the discrepancy between the contract quote P and the spot price S of the underlying asset. Specifically, we define the *multiplicative perpetual/spot basis* $B^{\mathrm{p}}_{\mathrm{m}}$ implicitly from this expression:

$$P = S \cdot (1 + B^{\mathrm{p}}_{\mathrm{m}}) \qquad \text{where} \quad 1 + B^{\mathrm{p}}_{\mathrm{m}} > 0\,. \tag{1}$$

Here the superscript $^{\mathrm{p}}$ reminds us that we are dealing with perpetual contracts so that we can distinguish it from the basis for futures contracts defined below. Similarly the subscript $_{\mathrm{m}}$ is a reminder of the fact that this is a multiplicative basis. More explicitly, consistently with definitions (1), the basis $B^{\mathrm{p}}_{\mathrm{m}}$ is defined as

$$B^{\mathrm{p}}_{\mathrm{m}} = \frac{P}{S} - 1\,. \tag{2}$$

Since both P and S are positive, the multiplicative basis $B^{\mathrm{p}}_{\mathrm{m}}$ can assume any value greater than -1. When the multiplicative basis $B^{\mathrm{p}}_{\mathrm{m}}$ is negative, the perpetual contract is quoted at a discount relative to the price. Conversely, when $B^{\mathrm{p}}_{\mathrm{m}}$ is positive, the contract is quoted at a premium relative to the

Basis Type	Multip. (bips)	Log. (bips)	1-Week Rate (%)
Perpetuals/Spot	-0.181061	-0.181063	-0.094410
F_1/Spot	-5.431831	-5.433306	-2.832312
F_0/Spot	-9.815104	-9.819924	-5.117876
Perpetuals/Futures	-9.634217	-9.638861	-5.023556

Table 2: Basis numbers and equivalent rates for different ratios, for perpetual/spot, nearest futures/spot, zero-time-to-expiry/spot, and perpetual/futures. Note that the basis numbers are expressed in basis points, while the equivalent rate is expressed in percentage points. The 1-week equivalent rate is defined in the text.

spot price. Equilibrium between the perpetual quote and the price is achieved when the basis is precisely zero.

In order to analyze the statistical properties of the basis numbers across a broad spectrum of observations it is advantageous to define the logarithmic basis as

$$B_\ell^{\mathrm{p}} = \log\left(\frac{P}{S}\right) = \log\left(1 + B_{\mathrm{m}}^{\mathrm{p}}\right) , \tag{3}$$

where the "log" here symbol represents the standard natural logarithm function and the subscript $_\ell$ in the symbol B_ℓ^{p} is a reminder that this is a logarithm-based number. Note that B_ℓ^{p} is defined across the entire real-number line. Therefore statistics of the logarithmic basis B_ℓ^{p} can be modelled, for example, using a Gaussian distribution. Also notice that, since for any small real number x we have

$$\log(1+x) \simeq x \qquad \text{for} \quad |x| \ll 1 ,$$

then for small values $B_{\mathrm{m}}^{\mathrm{p}}$ we have

$$B_{\mathrm{m}}^{\mathrm{p}} \simeq B_\ell^{\mathrm{p}} . \tag{4}$$

However, while $B_{\mathrm{m}}^{\mathrm{p}}$ is limited to be greater than -1, B_ℓ^{p} could in principle assume any real negative number. Finally we observe that the multiplicative basis $B_{\mathrm{m}}^{\mathrm{p}}$ and the logarithmic basis B_ℓ^{p} are either both positive or both negative.

The first row in table 2 shows the computation of the multiplicative and the logarithmic perpetual/spot basis for the BTC/USD, for the data in table 1. Note that, as explained by approximation (4), the values of the multiplicative and the logarithmic basis are very close to each other.

In order to make the interpretation of the basis numbers more straightforward, so that they can be compared with the projection rates defined below, we define the concept of the 1-week equivalent rate. For any given multiplicative basis number, we define the *1-week equivalent rate* as the constant rate that would be necessary to achieve the same percentage variation of the basis in one week. In the case of the reference data of table 1, $B_{\mathrm{m}}^{\mathrm{p}}$ is -0.181061 basis points and corresponds to a rate -0.094410%, which applied uniformly for one week to the spot price of 27,615.00 would result in the perpetual quote of 27,614.50.

Serenity publishes hourly both the multiplicative basis and the logarithmic basis for all available perpetual contracts.

3 Futures Contracts and the Projection Curve

Futures contracts are standardized agreements between two parties to buy or sell an asset at a predetermined price at a specified future date and time. Unlike spot trading, where the transaction and asset delivery occur immediately, futures contracts entail a commitment to complete the transaction at a later date. This mechanism allows traders and investors to hedge against price volatility and speculate on price movements without the immediate need for capital outlay or asset transfer.

Comparatively, perpetual contracts, as discussed in section 2, allow for similar speculative and hedging activities but without a predetermined expiration date. This key difference means that traders can hold their positions indefinitely in a perpetual contract, while in a futures contract, positions are settled at the contract's expiration date.

3.1 Projection rates

Arbitrage relationships play a pivotal role in deciphering the pricing and dynamics of financial instruments in the market. For instance, reference [1] elucidates a simple arbitrage-free relationship between the spot price and the forward price. Although theoretically sound, the derivation of this relationship often encounters deviations in digital asset markets, where arbitrage opportunities may arise. Therefore, we aim to provide a definition of projection rates that stands independently from any arbitrage relationship.

Threshold maturity hours

Consider a specific trading pair, for instance, Bitcoin/USD, and all the futures contracts associated with this pair on a given exchange like, e.g., Deribit. We focus on the futures contract with the shortest maturity. It's a common practice for traders to rollover their contracts to the subsequent maturity (i.e., the second shortest maturity) a few hours before expiration to circumvent delivery. We observed that quotes associated with maturities shorter than approximately 12 hours, tend to lose liquidity and eventually are removed from the list of actively traded futures. We introduce the term *threshold-maturity-hours* to denote the minimum number of hours required in the futures for a quote to be deemed valid. In the aforementioned scenario the threshold maturity hours, denoted with $T_{\min}$, is 12 hours. Cloudwall will periodically review the threshold maturity hours to ensure that the quotes utilized to bootstrap the projection rates always exhibit adequate liquidity.

Date/time conventions

As illustrated in Table 1, at any given moment and for the same specified trading pair, a variety of futures contracts with different maturities are available. Unlike traditional commodity futures, digital asset futures are traded 24 hours a day, 7 days a week. Therefore, when calculating the residual maturity, it is crucial to consider not only the number of days before expiry but also the exact time.

When the need arises to compute the elapsed period between two distinct points in time, we use the difference in the number of days, adding a fraction so that an hour is counted as 1/24 days, a minute is counted as 1/60 hours, and so on. The fractional number of days obtained in this manner is then divided by the factor 365, to obtain the year fraction between the two points in time.

Maturity	p_j	f_j	r_j
BTC-13-OCT-23	5.19%	5.19%	-6.43%
BTC-20-OCT-23	5.19%	5.19%	1.64%
BTC-27-OCT-23	3.30%	1.42%	1.55%
BTC-24-NOV-23	4.08%	4.47%	3.36%
BTC-29-DEC-23	5.00%	6.09%	4.56%
BTC-29-MAR-24	4.87%	4.75%	4.66%
BTC-27-SEP-24	5.22%	5.55%	5.12%

Table 3: Values of the projection, forward and spot rates computed using the futures quotes of table 1. Note that in the computation of the spot rates we used a price of 27,615.00.

3.2 Definition of projection rates

Suppose we have a collection of n futures contracts listed on a specific exchange, all based on the same underlying pair. These contracts are arranged in ascending order of their time to maturity, with corresponding quotes marked F_1, F_2, ..., F_n, and time-to-maturity in year fractions at $T_1 < T_2 < \ldots < T_n$. Note that we restrict our selection to contracts expiring after the minimum threshold hours, i.e. with $T_1 \geq T_{\min}$.

We define the n-1 continuously-compounded projection rates p_j as,

$$p_j = \frac{1}{T_j - T_1} \log\left(\frac{F_j}{F_1}\right) \qquad \text{for} \quad j = 2, \ldots, n\,. \tag{5}$$

While we are not making any arbitrage assumption in the definition of the projection rates, one could see the resemblance of our definition with the definition of forward-starting zero rates between T_1 and T_j, for $j = 2, \ldots, n$.

Given F_1 we can invert equation (5) to obtain the futures quotes in terms of the projection rates:

$$F_j = F_1\, e^{p_j(T_j - T_1)} \qquad \text{for} \quad j = 2, \ldots, n\,. \tag{6}$$

If we were to assume the projection rates to define some kind of zero rates between T_1 and T_j then they would reflect the market expectation for projecting F_1 from T_1 to T_j, for $j = 2, \ldots, n$ (which provides a justification for the *projection rate* nomenclature).

In the second column of table 3, we show the projection rates p_j's computed using the contract BTC-13-OCT-23 as the contract at T_1, for the dataset in table 1. Similarly in figure 1 we show a plot of the projection rates described in table 3. As evident from both the table and the figure, at the time we captured the market data, the forward prices shows a typical upward sloping curve of a *contango market*. As a consequence the corresponding projection rates are all positive.

3.3 Interpretation of the projection rate

Bitcoin is often referred to as "digital gold" due to its scarcity and store of value characteristics, similar to physical gold. For example, Ferdinando Ametrano, a notable figure in the cryptocurrency and blockchain space, has explored this comparison extensively in his work (see, e.g., reference [3]). Even though Bitcoin does not pay any dividend, it can be considered to have a convenience yield. Convenience yields are typically associated with commodities and represents the non-monetary benefits of holding an asset rather than a derivative contract on the asset itself. In the case of Bitcoin,

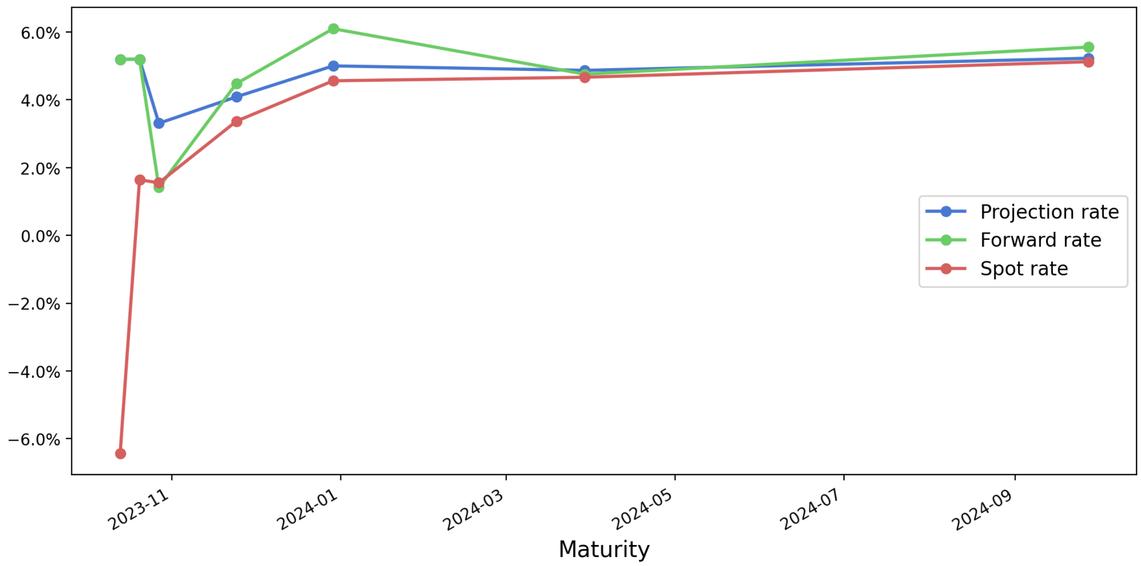

Figure 1: Plot of the projection, forward and spot rates presented in in table 3

the convenience yield could be interpreted as the benefit derived from holding a decentralized, borderless, and censorship-resistant form of money that provides a hedge against inflation and monetary policy changes. For other digital assets the convenience may come from other sources, for example in the case of the Ether token it may arises from the utility of paying gas to run smart contracts on the Ethereum blockchain. We assume that any digital asset has a certain convenience in holding it directly as opposed to entering into a contract for its future delivery and that this convenience is paid at a constant yield q.

In the conventional Black-Scholes option-pricing framework for a commodity with a continuously-paying convenience yield q in a market with a continuously compounded interest rate i, the forward-forward arbitrage relationship is given by:

$$F_b = F_a \cdot e^{(i-q)(T_b-T_a)} \quad \text{with} \quad T_b > T_a \,,$$

where F_a and F_b denote the quotes of two forward contracts, one maturing at T_a and the other at T_b. The term $i-q$ in the exponent signifies that the exponential growth is proportional to the difference between the interest rate i and the convenience yield q. Comparing this equation with equation (6) we observe that the projection rate can be thought as the difference between the risk-free rate and the convenience yield:

$$p = i - q \,.$$

Given the observations by reference [1] regarding the ambiguity in selecting the interest rates for digital assets, coupled with the unclear definition of a *convenience yield*, for digital tokens, we opt to calculate the projection rate as given, without further assumptions, and base the forward price computation directly on p.

3.4 Interpolation and extrapolation of projection rates

We previously defined the projection rates on maturities of listed futures contracts. A natural question arises: how do we extend this definition to cover maturities that are not listed on the market?

Several scenarios come to mind where we might be interested in knowing projection rates not matching the futures maturities, e.g.:

1. Estimate the forward price when its maturity does not match any of listed futures contracts.

2. Define a time-invariant term structure of constant-maturity projection rates. For example, every day compute the projection rates for tenors of 1-week, 1-month, 3-month, and so on.

Therefore we need to find the best way to interpolate between maturity dates of futures contracts. Moreover we need to be able to extrapolate the projection rates for maturities that are either preceding T_2 or succeeding T_n.

In general, given a maturity T, we would like to write the expected forward price as a function of the continuously compounded projection rate $p(T)$, so that

$$F(T) = F_1 \cdot e^{p(T)(T-T_1)}\,, \tag{7}$$

with $p(T)$ is defined for all T's.

Extrapolation of projection rates

Let's start with the extrapolation of the p_j's. We use constant flat extrapolation both before and after the observed rates. Therefore when extrapolating to dates that are earlier than the second-nearest futures maturity, we define:

$$p(T) = p_2 \qquad \text{for} \quad T_0 \leq T \leq T_1\,,$$

where we recall that T_0 is the year fraction corresponding to the current time. In particular we define

$$p_1 = p(T_1) = p_2\,.$$

Similarly, when extrapolating to dates that are later than the latest futures-contract maturity, we define:

$$p(T) = p_n \qquad \text{for} \quad T \geq T_n\,,$$

where we recall that T_n is the maturity of the longest-dated futures contract.

Interpolation of projection rates

The interpolation of the projection rates is a bit trickier, since we would like to use it in the largest numbers of quantitative models, such as the Black-Scholes-Merton model for pricing derivative assets (see reference [4]). On the other hand we do not want to use exceedingly complicated interpolation methods, such as those based on cubic splines (see, e.g., reference [2]).

Taking inspiration from the interest world, we interpolate linearly on the logarithm of the forward price, in each maturity segment. As shown in reference [2] this method corresponds to piecewise-constant forward curves. In practice take a time T so that for $T_{j-1} \leq T < T_j$ for some $j = 2, \ldots, n$, then define,

$$F(T) = F_{j-1} \cdot e^{f_j \dot{(}T-T_{j-1})}\,, \tag{8}$$

where, for consistency, f_j must be such that $F(T_j) = F_j$, i.e.:

$$F_j = F_{j-1} \cdot e^{f_j \dot{(}T_j-T_{j-1})}\,.$$

We can derive the expression for the forward rate f_j by inversion of this last equation:

$$f_j = \frac{1}{T_j - T_{j-1}} \log\left(\frac{F_j}{F_{j-1}}\right) . \tag{9}$$

Note that f_2=p_2, hence we also define f_1=f_2.

Table 3 provides the values for the forward rates f_j's for the market data of table 1. As expected, as it can also be seen in figure 1, the variability of the forward rates is higher than that of the projection rates.

If we take $F(T)$ from (7) and we substitute it into equation (8) we obtain:

$$F_1\, e^{p(T)(T-T_1)} = F_{j-1} \cdot e^{f_j (T-T_j)} .$$

By taking the natural logarithm of this expression we have that:

$$p(T)(T-T_1) = \log\left(\frac{F_{j-1}}{F_1}\right) + f_j (T - T_j) ,$$

which can be further simplified, by using equation (5) for $j-1$, to

$$p(T)(T-T_1) = p_{j-1}\,(T_{j-1} - T_1) + f_j (T - T_j) ,$$

which finally yields

$$p(T) = p_{j-1} \cdot \frac{T_{j-1} - T_1}{T - T_1} + f_j \cdot \frac{T - T_j}{T - T_1} \qquad \text{for} \quad T_{j-1} \leq T \leq T_j . \tag{10}$$

This equation, recursively in j, provides the functional from of the time-dependent projection-rate function $p(T)$ in terms of the projection rates p_j's and the forward rates f_j's.

Using equation (7) for the extrapolation and equation (8) for the interpolation of the forward prices we can compute the expected forward price at any future date. For example in figure 2 we show the interpolated forward prices corresponding to a number of tenors applied to the the evaluation date of 10 October 2023 at 6:00 UTC. Similarly in figure 3 we show the projection rates corresponding to the forward prices of figure 2. Note that these projection rates are no longer linked to specific maturity dates so that their observation at different evaluation dates can provide interesting information about the dynamics of projection rates.

4 Basis Rates for Futures Contracts

In section 2, we elaborate on the concept of basis for perpetual contracts. In this section, we expand on the same concept for futures contracts. The discussion for futures contract basis is important for scenarios such as:

1. Employing futures in a strategy alongside the spot price.
2. Incorporating futures in a strategy with a perpetual contracts.
3. In the absence of perpetual contracts, substituting a perpetual contract with the shortest-maturity futures.

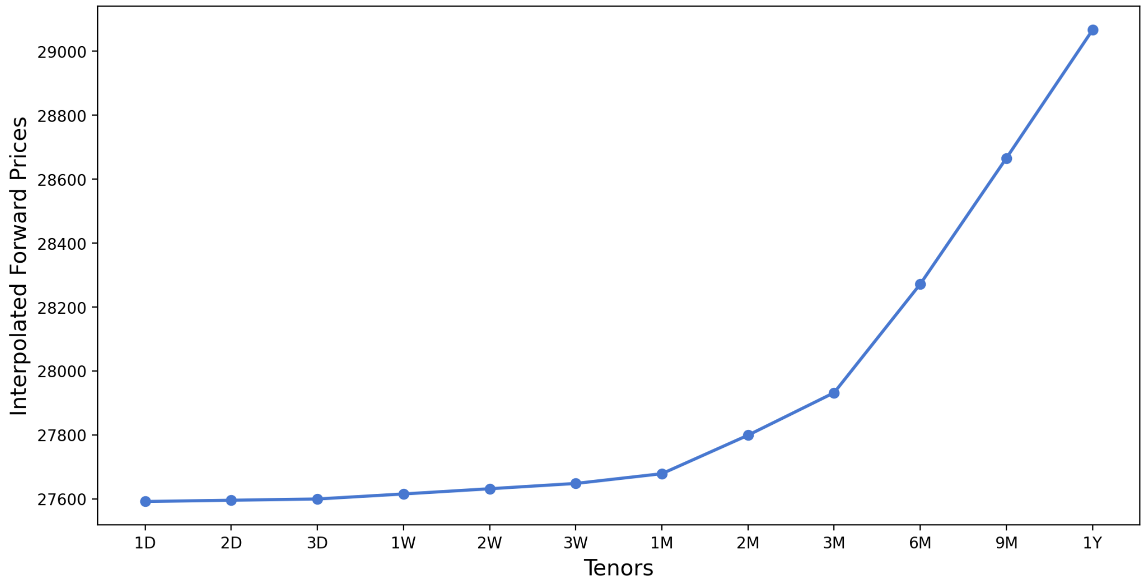

Figure 2: Interpolated forward prices computed using the data of table 3 for a number of fixed tenors applied to the evaluation date of 10 October 2023 at 6:00 UTC. Note that the tenor spacings are not drawn to scale of the actual time between nodes.

4.1 Nearest futures basis

Firstly we define the basis using the next expiry contract with maturity T_1, i.e. we define the *nearest-futures multiplicative basis* as

$$B_{\mathrm{m}}^{\mathrm{f1}} = \frac{F_1}{S} - 1\,. \tag{11}$$

Likewise, we define the *nearest-futures logarithmic basis* as

$$B_{\ell}^{\mathrm{f1}} = \log\left(\frac{F_1}{S}\right) = \log\left(1 + B_{\mathrm{m}}^{\mathrm{f1}}\right)\,. \tag{12}$$

In the symbol of basis the superscript $^{\mathrm{F1}}$ reminds us that the basis is defined using F_1 as reference quote. These definitions of basis are sensible, e.g., in a market where perpetual contracts are not quoted, and the nearest futures contract is used as a substitute.

By exponentiation of equation (12),

$$e^{B_{\ell}^{\mathrm{f1}}} = e^{\log(F_1/S)} = F_1/S\,, \tag{13}$$

we can obtain an expression for F_1 in terms of S and B_{ℓ}^{f1}:

$$F_1 = S e^{B_{\ell}^{\mathrm{f1}}}\,, \tag{14}$$

which is used below.

Note that it is not possible to compare different values of the nearest-futures basis at different dates, because the time to next expiry T_1 is not constant. We solve this problem in the next subsection by creating the zero-time-to-expiry futures basis numbers.

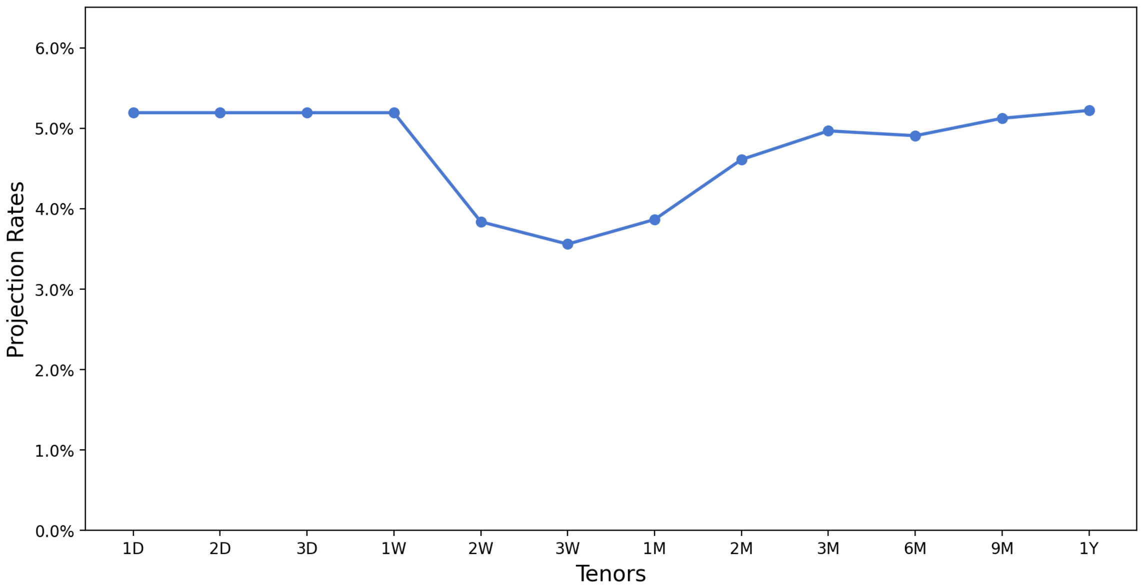

Figure 3: Term structure of interpolated projection rates corresponding to the forward prices of figure 2. Note that the tenors spacing are not drawn to scale of the actual time in between.

4.2 The zero-time-to-expiry futures basis

In an attempt to remove the varying time-to-next-expiry T_1, we project the first-maturity quote F_1 to the current time T_0. Therefore we define the zero-time-to-expiry futures virtual quote F_0 as,

$$F_0 = F_1\, e^{-p_1(T_1 - T_0)}\,, \tag{15}$$

where we recall that, by definition, $p_1 = p_2$. The value of F_0 can be viewed as the quote of an hypothetical futures contract that is about to expire immediately. In a perfectly efficient market with no arbitrage, the value of F_0 should be close to the spot price S:

$$F_0 \simeq S\,.$$

In reality this isn't always the case and there might exist persistent arbitrage opportunities (these opportunities are typically utilized by traders to make profit out of this imbalance). Therefore we define the zero-time-to-expiry futures basis measuring the differences between F_0 and S. Hence, the *zero-time-to-expiry futures multiplicative basis* $B_{\mathrm{m}}^{\mathrm{f0}}$ is defined as:

$$B_{\mathrm{m}}^{\mathrm{f0}} = \frac{F_0}{S} - 1\,. \tag{16}$$

Likewise, we define the *zero-time-to-expiry futures logarithmic basis* as

$$B_{\ell}^{\mathrm{F0}} = \log\left(\frac{F_0}{S}\right) = \log\left(1 + B_{\mathrm{m}}^{\mathrm{f0}}\right)\,. \tag{17}$$

Table 2 show both the multiplicative and the logarithmic basis for both the next-maturity/spot and the zero-time-to-expiry/spot basis. As a comparison we note that the 1-week equivalent rates are about 2.83% for the F_1/Spot basis and about 5.12% for the F_0/Spot basis. These numbers are significant and express that there is a possible arbitrage opportunity between the spot and the futures market.

4.3 Basis rates between perpetual and the futures contracts

So far we have defined the basis numbers either from futures or perpetual quotes with respect to the spot price. There is however a number of strategies that are completely independent of the spot price. For example one could devise a strategy using long positions in futures contracts at different maturities and hedge it using a short position in a perpetual contract.

Therefore we define the *perpetual/futures multiplicative basis* as

$$B_{\mathrm{m}}^{\mathrm{pf}} = \frac{F_0}{P} - 1\,, \tag{18}$$

similarly we define the *perpetual/futures logarithmic basis* as

$$B_{\mathrm{m}}^{\mathrm{pf}} = \log\left(\frac{F_0}{P}\right)\,. \tag{19}$$

In the above definitions of the perpetual/futures basis we use the zero-time-to-expiry futures virtual quote F_0 in order to remove possible variable time-to-maturity dependencies from the nearest maturity T_1.

Table 2 show both the multiplicative and the logarithmic basis for the F_0/perpetual basis. As a comparison we note that the 1-week equivalent rate of about 5.02% also indicates a significant possible arbitrage opportunity between the perpetual and the futures market.

5 Spot-based projection rates

In this section we compare the above definition of the projection rates p_j, as defined in equation (6), with other formulations that are based on the relationship between futures quotes and the spot-market price.

Consider for example the spot-based rates defined in reference [1]:

$$F_j = S \cdot e^{r_j(T_j - T_0)} \qquad \text{for} \quad j = 1, 2, \ldots, n\,. \tag{20}$$

Note that this definition provides n rates, while our previous definition of p_j only provides n-1 rates. (As we shall see shortly, the *missing parameter* is the nearest-futures basis.)

Since, for $j = 2, \ldots, n$, both equation (6) and (20) are expression for F_j, we can eliminate F_j from them to obtain:

$$F_1 \cdot e^{p_j(T_j - T_1)} = S \cdot e^{r_j(T_j - T_0)}\,.$$

Now we use equation (14) for F_1 to obtain,

$$S \cdot e^{B_\ell^{\mathrm{f1}}} \cdot e^{p_j(T_j - T_1)} = S \cdot e^{r_j(T_j - T_0)}\,,$$

that can be further simplified by eliminating S from both sides:

$$e^{B_\ell^{\mathrm{f1}}} \cdot e^{p_j(T_j - T_1)} = e^{r_j(T_j - T_0)}\,.$$

Taking log on both sides of this expression and dividing by the maturity term $T_j - T_0$ gives us:

$$r_j = \frac{B_\ell^{\mathrm{f1}}}{T_j - T_0} + p_j \cdot \frac{T_j - T_1}{T_j - T_0} \qquad \text{for} \quad j = 2, \ldots, n\,, \tag{21}$$

i.e. an expression for the spot-based projection rates in terms of the nearest-futures log basis and the projection rates defined in equation (5).

Therefore, we have just demonstrated that it is equivalent to use the n spot-based rates r_j, for $j = 1, \ldots, n$, or the n-1 projection rates p_j together with the basis B_ℓ^{f1}.

Since the only dependence on the spot price in equation (21) is brought by the basis term B_ℓ^{f1}, we can deduce that the rates r_j will always have a residual dependence on S. Therefore, it is advisable to use the r_j rates when we want to highlight the dependence on the spot price of a given asset or portfolio. However, when our strategy is not directly dependent on the spot price, such as in a futures-only portfolio, it is advisable to use the projection rates p_j so that the spot-price dependence is eliminated a priori.

For traders and portfolio managers who wish have to have a risk view based only on market future expectations, the ability to isolate the spot effect can be crucial.

Figure 1 shows the spot rates together with the projection rates. We notice that the first term on the right-hand-side of equation (21) is responsible for *bringing down* the whole spot-rate curve with respect to the projection curve. This negative effect can be measured at the one-week spot by the 1-week equivalent rate of $B_{\mathrm{m}}^{\mathrm{f1}}$ which, as shown in table 2, is about -5% .

6 Conclusions

This study explored the dynamics of financial instruments in digital asset markets, focusing on perpetual and futures contracts. The basis for perpetual contracts was examined by comparing the spot price and perpetual contract quotes. The discussion then moved to futures contracts, introducing the concept of projection rates to understand price dynamics across different maturity points. The computation of basis for futures contracts is also explained, covering scenarios of nearest and extrapolated-futures basis. These analyses shed light on the relationship between spot prices, perpetual quotes, and futures quotes, enriching the understanding of market dynamics. The methodologies presented provide a foundation for further analysis, aiding in the development of informed trading strategies in the digital asset domain.

In conclusions, the Serenity software by Cloudwall significantly aids portfolio managers in handling linear derivatives, such as perpetual and futures contracts in the digital asset market. It provides in-depth market data and analytical tools for understanding derivative dynamics, crucial for effective risk management.

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