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ANALYSIS

Delta Hedging for Digital Asset Options

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Sonakshi Rohra, Quantitative Researcher, Cloudwall*

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Abstract

Delta hedging often plays a central role in managing directional risk for options traders. However, the Black-Scholes model, which is often used to compute option delta and other greeks, involves assumptions that are usually not reflective of real-world market conditions, especially in the highly volatile digital assets market. This paper introduces Serenity's smile-adjusted delta as a subtle approach that accounts for the volatility smile phenomenon observed in options markets. We follow with an empirical analysis of delta hedging strategies for Bitcoin options, focusing on Serenity's smile-adjusted delta compared to the traditional Black-Scholes delta. Our study recognizes the unique characteristics of the digital asset market and uses Serenity's proprietary derivatives analytics and volatility surfaces to enhance risk management techniques.

1 Introduction

Among the diverse instruments within the digital assets space, options have gained prominence as essential tools for investors and traders seeking to mitigate risk and enhance returns. Delta hedging, a widely adopted strategy, plays a pivotal role in managing risk associated with options by dynamically adjusting portfolio positions to maintain delta neutrality. In particular, it manages the directional risk of the option position by trading delta units of the underlying asset. This ensures that the value of the portfolio remains unchanged when small changes occur to the value of the underlying. However, the computation of the delta itself is a subject of interest, and remains highly model dependent.

The Black-Scholes model famously comes to mind when we look for closed form solutions for option pricing and hence, closed form solutions for delta computation. However, it is important to note that while the Black-Scholes model provides a valuable framework for option pricing and delta hedging, its assumptions might not perfectly reflect real-world market conditions. In particular, delta hedging under Black-Scholes assumes perfect hedging if the volatility associated with the underlying asset is non-stochastic, there are no transaction costs and the hedging is done continuously, all three of which are impractical and unrealistic assumptions.

Moreover, moves in volatility are often correlated to spot moves, which remain unaccounted for due to Black-Scholes' treatment of volatility held constant and independent. In essence, this assumption of constant volatility ignores the presence of fat tails or non-normally distributed behaviour in the returns distributions of assets, which certainly is not the reality in the digital assets space. Hence, the intricate nature of the digital asset market, characterized by high volatility and unique market dynamics, necessitates a nuanced approach to delta hedging.

At Cloudwall, we use a Stochastic Volatility Inspired (SVI) - parametrised volatility surface to model the behaviour of Implied Volatility. Refer to [1] for a detailed treatment of our state-of-the-art volatility surfaces. In this paper, we undertake a comprehensive comparative analysis of two prominent delta hedging strategies employed in the context of Bitcoin options: smile-adjusted deltas

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computed using Cloudwall's in-house Volatility Surfaces and Analytics, and the classic Black-Scholes deltas. Smile-adjusted deltas account for the volatility smile phenomenon often observed in options markets, acknowledging the non-uniform distribution of implied volatilities across different strike prices.

The motivation behind this study stems from the need for advanced risk management strategies that cater to the distinct characteristics of digital asset options. Bitcoin, as a pioneer in the digital assets realm, presents an intriguing case study due to its high volatility and market behavior that may differ from traditional financial assets. At Cloudwall, we find that the digital asset market's relatively nascent nature, coupled with its rapid developments, calls for a deeper understanding of effective risk mitigation techniques.

To achieve this, our analysis will span multiple dimensions. We analyse some implied volatility dynamics for BTC options over the year 2022, followed by a comparison of modelling choices for option greeks in Serenity* to the standard Black-Scholes model. Further, we quantitatively assess the performance of both delta hedging strategies using historical Bitcoin prices over 2022 and implied volatility data. Additionally, we will delve into the implications of varying rebalancing frequencies on the effectiveness of these strategies.

1.1 Behaviour of Implied Volatility

The Black-Scholes model for option pricing involves six parameters to price options, one of which is the implied volatility, or the estimate of future variability in the underlying asset price, as implied by the market. In mathematical terms, the implied volatility σ_{imp} is the value of σ for which the theoretical option price given by the Black-Scholes model equals the listed option price.

$$C(S,K,T) = V(S,r,p,\sigma_{imp};K,T,\phi) \tag{1}$$

Implied volatility modelling has been crucial to derivatives modelling ever since the Black-Scholes model was criticised for its flat volatility assumption. While this makes for a simplified model, this is far from reality, where volatility is indeed stochastic in nature. Moreover, option prices, like any other asset, are determined by market supply and demand. Given that options are largely used for hedging and speculative purposes, it is not surprising that demand for OTM and ITM options are higher. In turn, the volatility for these options are also generally higher relative to ATM options and this gives rise to the phenomenon called a *volatility smile*.

Over the course of 2022, the behavior of the volatility smile for Bitcoin options showcased intriguing patterns that reflected the prevailing market dynamics. Figure 1 illustrates the empirical behaviour of the implied volatility of a BTC option expiring in December 2022, over four significant dates for digital assets in 2022: The Wormhole bridge hack, the Terra LUNA crash, the Ethereum Merge in July and the FTX Crash during mid-November.

In most instances, particularly during events like the Wormhole bridge hack and the Terra LUNA crash, we note a prevailing negative skew pattern for implied volatility. In these cases, implied volatility tends to be higher for out-of-the-money (OTM) puts than for OTM calls. Additionally, the Ethereum Merge event indicates a transition toward a more neutral volatility smile, signifying a potential shift toward positive market sentiment and a greater demand for OTM calls.

As news of the FTX crash broke and the cryptocurrency markets experienced a rapid and substantial

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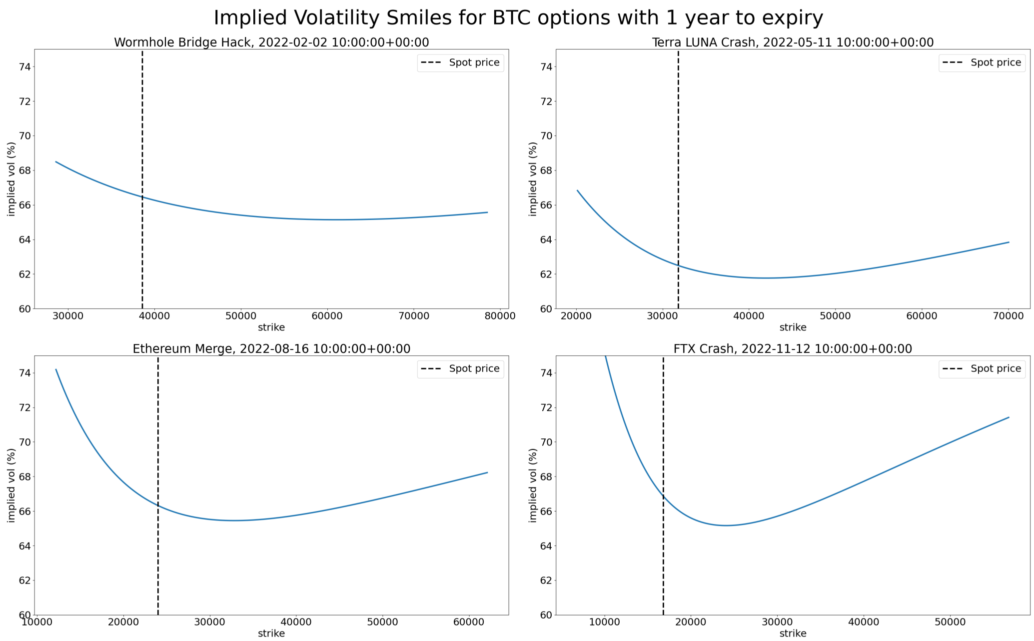


Figure 1: Behaviour of Implied Volatility for a BTC option

decline in prices, the volatility skew underwent a notable transformation. Deep out-of-the-money (OTM) put options, which provide protection against price declines, saw a remarkable surge in implied volatilities. This increase reflected a heightened demand for downside protection as uncertainty and panic swept through the market. We also observe a shrinking of the smile during the FTX crash, indicating lower demand for at-the-money contracts, which is unsurprising.

Overall, we find empirical evidence confirms that volatility for digital assets follow a stochastic process and are driven by movement in the underlying asset itself. We find that in the case of Bitcoin, the behaviour of implied volatility signaled the market's awareness of tail risks, given the presence of the skew during periods of uncertainty. Such patterns reveal that traders and investors are placing more emphasis on protecting against potential extreme price movements. Moreover, this movement in volatility is not linear (i.e., the volatility skew is a nonlinear function of change in underlying). This suggests that traders and investors anticipate different levels of volatility for options that are deep in-the-money (ITM) or deep out-of-the-money (OTM).

2 Options Delta in Serenity

The theoretical convention for (Black-Scholes) delta is that we have a unique value for delta, given a set of parameters. However, in reality the trading delta is computed based on *rules of thumb* or modelling choices. To account for the stochastic behaviour of volatility in particular, we are concerned with those choices that deal with changes in the volatility surface over time.

2.1 The Sticky Strike Rule

The sticky strike rule of thumb assumes that the implied volatility σ is independent of changes in the spot (i.e., the implied volatility smile doesn't change for change in the spot price) while "sticking" to a specific strike price. Mathematically, we say that the implied volatility, when expressed as a function of strike K, is invariant in the spot spot price S. if $\sigma(S; K, T)$ is the implied volatility for given strike K and maturity T, then the sticky strike rule is the same as:

$$\sigma(S + dS; K, T) = \sigma(S; K, T) \Leftrightarrow \frac{\partial \sigma(S; K, T)}{\partial S} = 0 \tag{2}$$

In this case, the volatility surface is determined by the time to maturity T and strike K, and we can write the implied volatility for a specified option as $\sigma(K, T)$. Then, the value of the option can be expressed as:

$$C(S, K, T, \sigma(K, T)) = V(S, r, p, \sigma(K, T); K, T, \phi) \tag{3}$$

The option price sensitivity to the underlying is simply given as

$$\Delta_{BS} = \frac{\partial C}{\partial S} \tag{4}$$

This is an attractive assumption because it enables the Black-Scholes formula to be used to calculate the delta with the volatility parameter set to the implied volatility for strike K. Thus, it acknowledges the varying implied volatilities across different strike prices without assuming a constant volatility across strikes. The same is true of the gamma, which becomes $\frac{\partial^2 C}{\partial S^2}$ or the rate of change of delta with respect to the underlying.

The Black-Scholes delta used for this study is given by:

$$\Delta_{BS} = \frac{\partial C(S, K, T, \sigma(K, T))}{\partial S} = N\Big(\frac{\ln(S/K) + (r + \sigma(K, T)^2/2)T}{\sigma(K, T)\sqrt{T}}\Big) \tag{5}$$

where $N(\cdot)$ is the normal cumulative standard normal distribution function, C denotes the Black-Scholes call option pricing formula, S denotes the underlying BTC price, r is the discount rate and T is the time to maturity of the option.

2.2 The Sticky Moneyness Rule

The sticky moneyness rule of thumb assumes that the implied volatility, expressed as a function of spot, *moneyness* and maturity is invariant in the spot price S. For a given moneyness $m = \frac{K}{S}$, we define the implied volatility in the moneyness space as $\sigma(S; \hat{m}, T)$

$$\hat{\sigma}(S + dS; m, T) = \hat{\sigma}(S; m, T) \Leftrightarrow \frac{\partial \hat{\sigma}(S; m, T)}{\partial S} = 0 \tag{6}$$

where $\hat{\sigma}(S; m, T)$ is the same as $\sigma(S; K = mS, T)$ in the strike space.

Then the implied volatility of an option of a given maturity is modelled as a function of both, underlying S and strike K, i.e., $\sigma(S; K, T)$ in the absolute strike space. While the sticky strike remains attractive and provides ease for computation, the sticky moneyness rule is much more representative of reality, where the underlying influences the implied volatility and they are not independent of each other. In this case, the value of the option can be expressed as:

$$C(S, K, T, \sigma(S, K, T)) = V(S, r, p, \sigma(S, K, T); K, T, \phi) \tag{7}$$

Using the chain rule, it is easy to see that the delta of the option can now be expressed as:

$$\Delta_{SmileAdj} = \frac{\partial C}{\partial S} + \frac{\partial C}{\partial \sigma} \cdot \frac{\partial \sigma}{\partial S} \tag{8}$$

Note that the first term in the expression is the sticky strike delta (or the delta calculated using Black-Scholes) with the volatility parameter set equal to the implied volatility. In the second term, the vega of the option $\partial C/\partial \sigma$ is positive. On the other hand, $\partial \sigma/\partial S$ measures the sensitivity of the volatility to the spot. Given the payoff structure of the call option, it usually follows that if $\sigma(S, K, T)$ is a declining (increasing) function of the strike K, then it is an increasing (decreasing) function of spot S and Δ is greater than (less than) that of the Black-Scholes delta. As such, this delta approximately accomodates for the smile risk arising from shifts in the implied volatility due to its dependence on the underlying spot and this delta is called the *Smile Adjusted Delta*.

Serenity reports this smile-adjusted delta, using our in-house SVI volatility surfaces to infer $\sigma(S, K, T)$. A finite difference scheme is used to compute the differentials instead of the Black-Scholes formula. Using the central difference approximation, we have:

$$\Delta_{Serenity} = \frac{V(S^+, r, p, \sigma(S^+, K, T); K, T, \phi) - V(S^-, r, p, \sigma(S^-, K, T); K, T, \phi)}{S^+ - S^-} \tag{9}$$

Where $S^\pm = S(1 \pm d)$ for $d = 0.01$ and the parametrised volatility $\sigma(S^\pm, K, T)$ is extracted from Serenity's SVI implied volatility surface.

3 Methodology for Delta Hedging

To examine the delta hedging performance of both, the Black-Scholes delta and the Serenity delta, we construct a self-financed delta-hedged portfolio with one short position in the BTC European-call option and Δ units of the underlying BTC. We also simulate a money market account, M_t which grows at the discounting rate r. Assuming continuous compounding, the money market account grows at a rate of $e^{\frac{r_t}{365}}$ for annualised discounting rate at time t, r_t.

Let Π_t represent the value of the hedging portfolio at time t. At time $t = 0$ we have:

$$\Pi_0 = \Delta_0 S_0 + M_0$$
$$M_0 = V_0 - \Delta_0 S_0$$

Since Π is self-financing, the money market account at time 0 consists of the positive option premium from the short position, and the negative value of Δ units of the underlying.

At the next time step, the portfolio is rebalanced in order to remain delta-neutral. The value of Δ_0 units of the underlying is now $\Delta_0 S_1$. We recalculate the new delta at this time step to be Δ_1 and the new amount of the underlying asset that is needed in the portfolio using the following day's delta and underlying BTC price.

Then, the additional amount of the underlying to be purchased is given as:

$$(\Delta_1 - \Delta_0)S_1$$

If the amount that is required in the underlying asset increases, we withdraw money from the money market account in order to purchase the supplementary amount needed of the underlying asset. On the other hand, if the amount in the underlying asset decreases, we sell the surplus and place the proceeds in the money market account.

At the same time, the money market account earns interest overnight at the discount rate for time t. Thus, the new money market account at time 1 is given as the sum of the gains or losses incurred from our position in the underlying and the interest earned from the money market accont:

$$M_1 = M_0 e^{r_0/365} - (\Delta_1 - \Delta_0)S_1 \tag{10}$$

Before we rebalance our position [1], we have $\Pi_1^- = \Delta_0 S_1 + M_0 e^{r_0/365}$

Since the portfolio Π replicates the option, the absolute value of the difference between the replicating portfolio and the value of the option represents the hedging error: $hedging\ error_t = \epsilon_t = V_t - \Pi_t^-$

Since our main objective is to minimise the directional risk associated with the long option, we analyse overall performance of the delta-hedging strategy using the root mean squared hedging error, given by:

$$RMSHE = \sqrt{\frac{1}{n}\sum_{t=1}^{T} \epsilon_t^2} \tag{11}$$

This is used as the primary evaluation criteria.

4 Data

The data used in this study contains prices of BTC options traded on Deribit, which captures a large segment of the crypto derivatives market with an aggregate daily traded volume of 550bn at the time of writing this article. Such large trading volumes seen on Deribit makes it the most attractive exchange for derivatives research in the digital assets space. Moreover, unlike CME (which also offers options on BTC, but remains closed on weekends and holidays), Deribit's trading platforms are open 24/7. In this study, the European-style call options on BTC are used since these capture the largest market and are liquid enough. We use daily call option prices and greeks, as well as the discounting and projection rate computed using Serenity's derivatives analytics APIs over the period 01-Jan-2022 to 11-November-2022.

[1] After we rebalance our position, we have $\Pi_1^+ = \Delta_1 S_1 + M_1$

The context of 2022 is particularly intriguing. It marked a period of both challenges and opportunities for the cryptocurrency market. Regulatory developments, institutional adoption, macroeconomic trends, and technological advancements shaped the landscape in ways that impacted options trading. The volatile price fluctuations and the continued emergence of innovative financial products make this year a critical case study for understanding the implications of delta hedging techniques.

Summary statistics for the sample are reported in Table 1. We report the statistics by segregating the data into three moneyness categories. A call option is said to be Out-of-The-Money (OTM) if the moneyness ratio $\frac{S_t}{K}$ is less than 0.98, At-the-Money (ATM) if the ratio is between 0.98 and 1.02, and In-the-Money (ITM) if the ratio is greater than 1.02. We report statistical behaviour of the spot price for BTC, call option price and the implied volatility for strike $K = \$22000$

Statistics	BTC spot	Call option	Implied Volatility
mean	37407.21	18240.85	0.7482
std	7373.12	7027.68	0.0278
min	22493.47	3810.27	0.6771
max	47621.73	29364.55	0.8893

Table 1: Summary statistics for when BTC option is In-the-Money(ITM)

Statistics	BTC spot	Call option	Implied Volatility
mean	21873.57	3883.13	0.7247
std	255.61	501.37	0.0150
min	21599.07	3125.73	0.7008
max	22198.64	4465.09	0.7449

Table 2: Summary statistics for when BTC option is At-the-Money(ATM)

Statistics	BTC spot	Call option	Implied Volatility
mean	19497.92	1995.42	0.6691
std	1496.28	1301.97	0.0718
min	15775.44	74.26	0.4906
max	21522.00	4717.51	0.8756

Table 3: Summary statistics for when BTC option is Out-of-the-Money(OTM)

5 Empirical Results

Holding maturity constant, the comparison of the Black-Scholes delta and the smile adjusted delta in Serenity is presented in Figure 2.

Note that the vega for a long call option is always positive. Thus, the relative difference between the black-scholes and the smile-adjusted delta is a result of the sensitivity of the implied volatility to the stock price, and thus indirectly a result of the slope of the volatility smile.

Figure 3 shows the volatility smile for the date 30-November-2022 and its corresponding slope with respect to *strike*. Note that the smile for this date is slightly negatively skewed, as the ATM Strike does not achieve the global minima of the smile, which occurs a higher strike, showing greater demand for put options than calls at this time. Indeed, the negative slope for OTM put strikes and

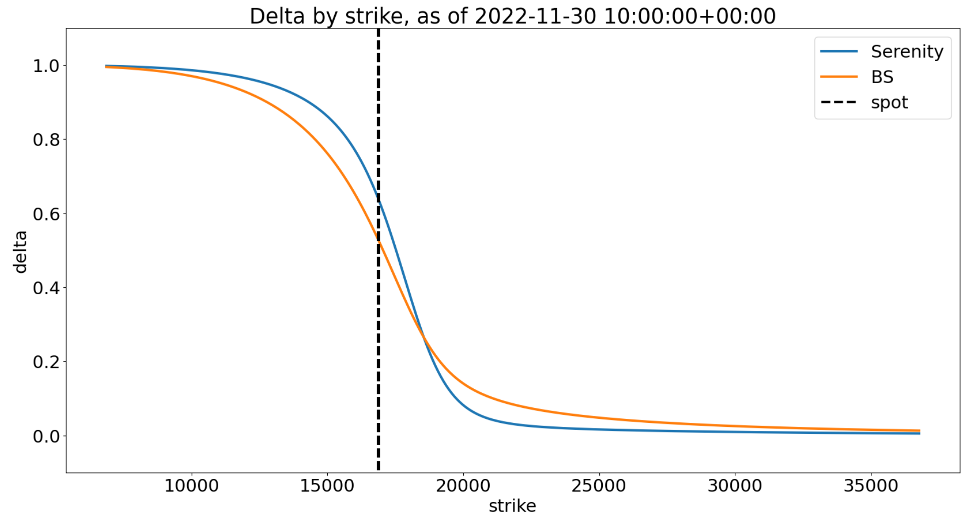
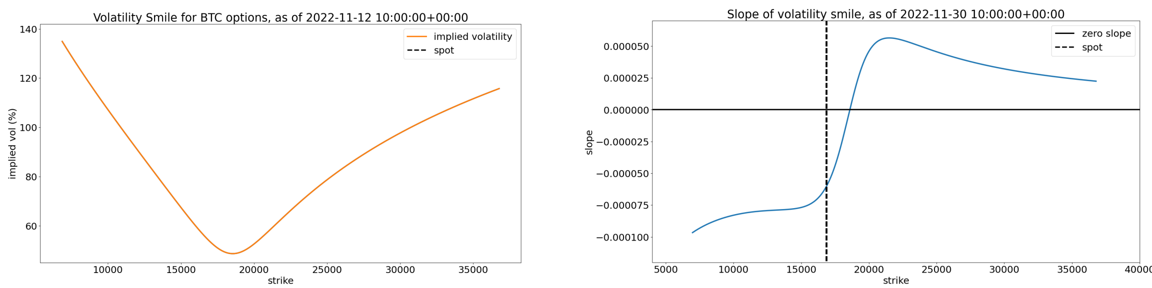


Figure 2: Comparison of Serenity's delta to the Standard Black-Scholes delta for long BTC options



(a) Volatility smile for BTC call options

(b) Slope of volatility smile (with respect to strike K)

Figure 3: Implied volatility dynamics with respect to strike K

some proportion of ITM (as seen from Figure 3b) put strikes largely influence the smile-adjusted delta which is higher than the black-scholes delta over these strikes.

Moreover, the smile-adjusted delta is not only influenced by the slope of the volatility smile but also the *vega*, shown in Figure 4. Since the vega is largest for ATM options, the difference between the two deltas is greatest around the ATM strike, marked by the black dashed line. Moreover, the difference between the deltas approaches zero for deep ITM and deep OTM options since the vega for these options is close to zero.

The two deltas are equal when the slope of the implied volatility smile is zero. This makes intuitive sense, since the Black-Scholes model assumes a flat volatility smile (i.e., zero slope) and in this case, the smile-adjusted delta is simply the same as the Black-Scholes delta. In other words, in stable market conditions, when volatility is independent of moves in the underlying asset price moves, the smile adjusted delta should be approximately equal to the Black-Scholes delta.

To compare the stability of the two deltas, we graph the delta profiles for an option expiring on December 29, 2022. As depicted in Figure 5, both the Black-Scholes and smile-adjusted deltas

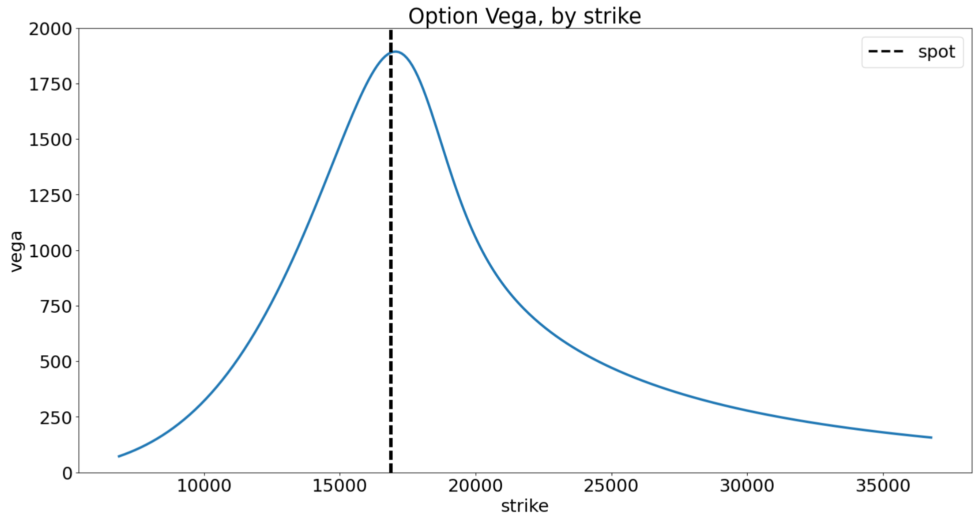


Figure 4: Option Vega for long BTC call options in Serenity

exhibit a parallel movement, albeit with a noticeable variance in magnitude. We note that the smile adjusted delta is usually higher than the Black-Scholes delta. Thus, a comparison using a delta hedging strategy for these two deltas studies the impact of the size of delta for market conditions of 2022.

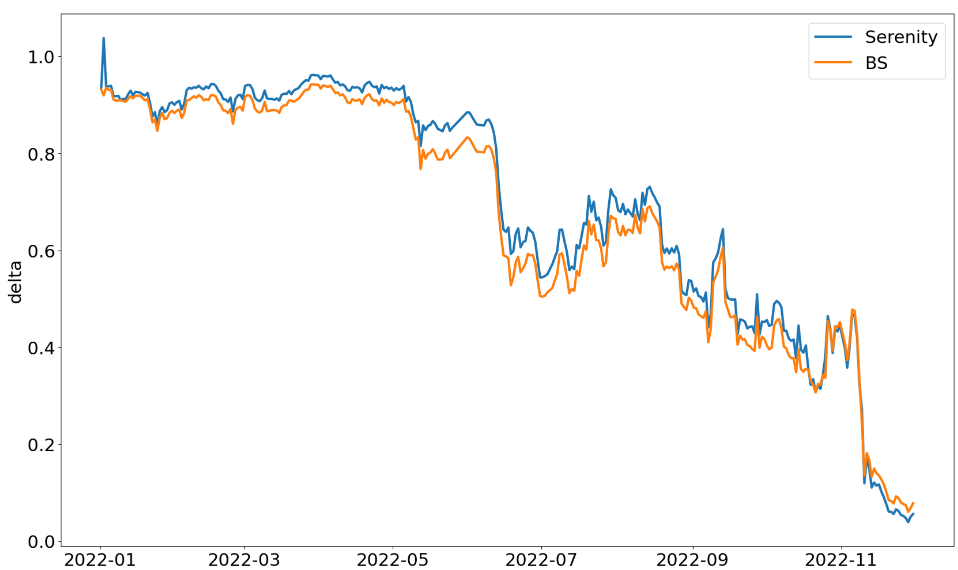


Figure 5: Comparison of the Black-Scholes and smile adjusted delta, by time

A smile adjusted delta that is higher than the Black-Scholes delta suggests that the BTC option's risk exposure to changes in the underlying asset is greater when we account for the volatility smile effect, compared to the traditional Black-Scholes model. Moreover, it suggests that the options market is assigning a larger sensitivity to changes in BTC's price and that market participants are pricing in the potential for more extreme price movements or tail risk that is not captured by the normal distribution assumption of Black-Scholes. Consequentially, a smaller amount of BTC is required to hedge each short position in the option as compared to the black-scholes world, due to the greater sensitivity to changes in the underlying asset.

Initially, we study the results of the delta hedging experiment on a daily hedging frequency. Subsequently, we analyze the effects of altering the frequency of rebalancing. The hedging error reflects the differences between the predicted changes in the portfolio value and the actual changes that occur due to fluctuations in the market.

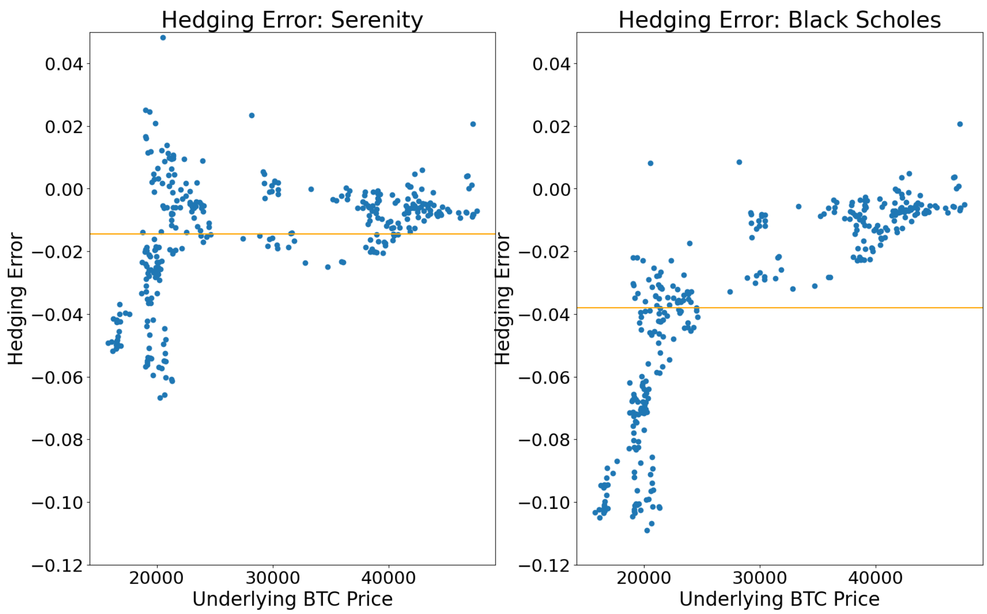


Figure 6: Comparison of the hedging error for Black-Scholes and smile adjusted delta, daily hedging

Figure 6 depicts the hedging error for an option expiring on December 29, 2022, when we rehedge the option daily. We find that for daily hedging frequency, the Black-Scholes delta performs worse than Serenity's smile adjusted delta. The average hedging error for the Black-Scholes delta is -0.03791, while the average hedging error for the Serenity delta is -0.01450. As such, the Serenity delta offers a more precise replication of the option's behavior, on average.

Furthermore, the hedging error associated with the Black-Scholes delta displays greater variability, characterized by a variance of 3.3267%. In contrast, the variance linked to the hedging error of Serenity's delta is approximately 1.8647%. Although, in both cases the average hedging error is negative, which means that on average, the actual hedging strategy is less effective than predicted. We also note that the variance of the hedging error is particularly large in bearish scenarios, when the underlying BTC price is lower than $< \$25,000$. Particularly in this bearish price range, the Black-Scholes delta showcases considerable negative hedging error values, whereas Serenity's delta performs considerably better.

Now, we check how our replication strategy performs when we adjust the rehedging frequency. We do this by keeping our delta constant until the next rebalance, at which we recompute the delta and so on. Figures 7 and 8 shows the hedging error for Serenity's and Black-Scholes delta over a range of 1, 3, 7, 10, 15 and 30 day rehedging frequencies respectively.

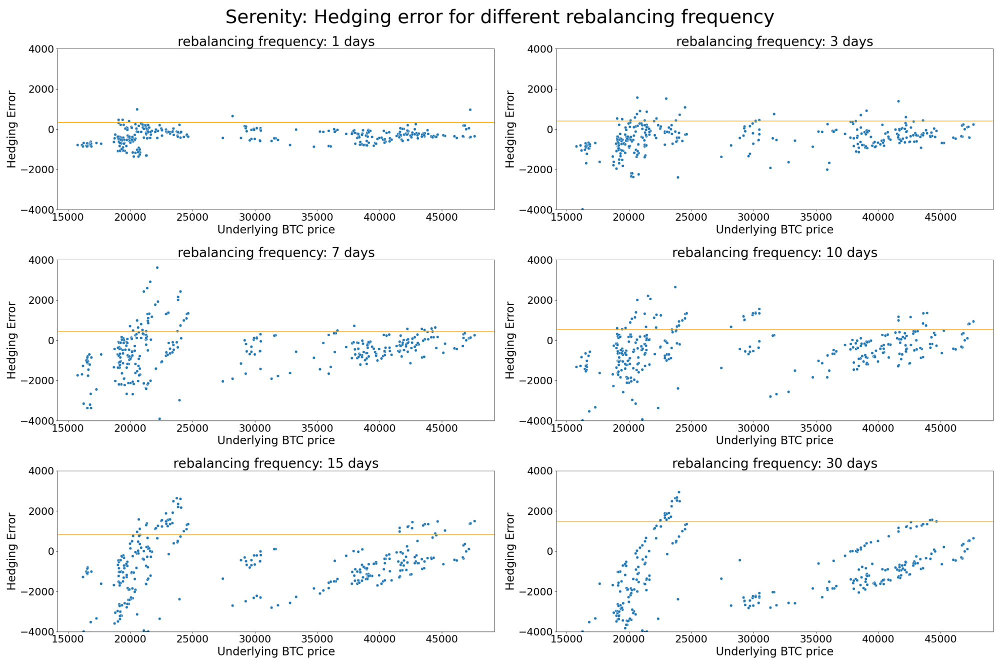


Figure 7: Comparison of the hedging error for Serenity delta for varying hedging frequency

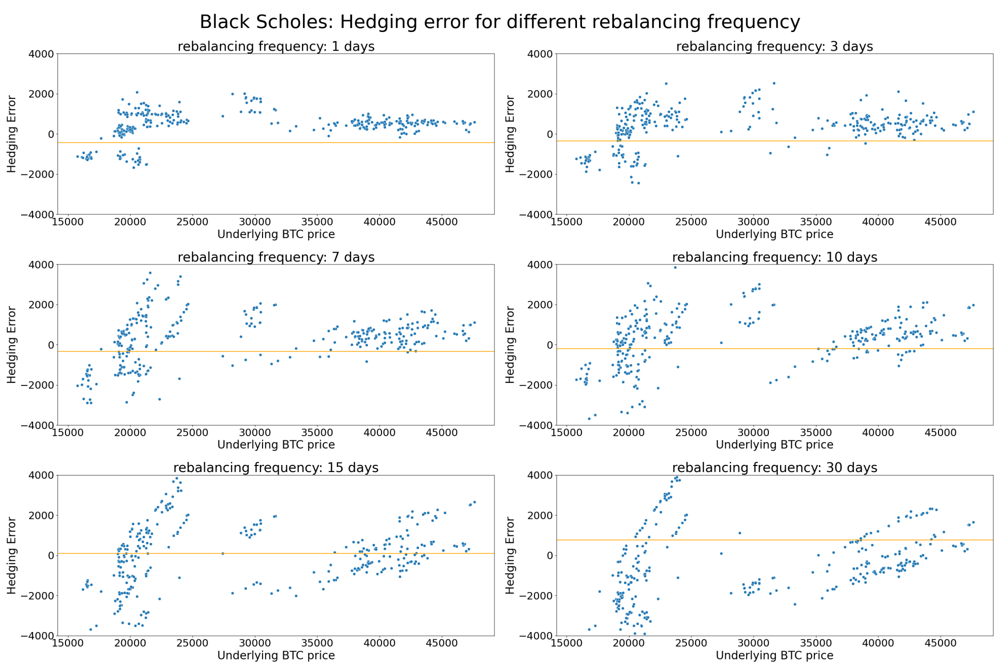


Figure 8: Comparison of the hedging error for Black-Scholes delta for varying hedging frequency

We find a consistent trend for both deltas: the more frequent the hedging, the lower and less varied in our hedging error. This is because when rehedging is done very frequently, the hedge is adjusted more often to reflect the changes in the underlying asset's price and implied volatility. Such frequent adjustment helps the portfolio stay closer to a delta neutral position in response to the market movement over time. In other words, the presence of gamma indicates that the delta is not constant with change in underlying and so the it needs frequent adjusting.

To conduct a formal comparison, we utilize the root mean square hedging error (RMHSE) calculated using equation 11 for both Serenity and Black-Scholes deltas across varying rehedging frequencies, as illustrated in table 4.

Hedging Frequency (in days)	$\Delta_{Serenity}$	Δ_{BS}	Difference ($\Delta_{Serenity} - \Delta_{BS}$)
1	3.404131	5.771409	-2.367278
3	5.479241	6.678476	-1.199235
7	7.878473	8.495061	-0.616588
10	9.009913	9.449138	-0.439225
15	12.280269	11.770574	0.509695
30	17.160948	15.578776	1.582171

Table 4: RMHSE for Delta hedging using Serenity and Black-Scholes delta over different rehedging frequencies

Analyzing the RMHSE results confirms the trends visualized in figures 7 and 8: The performance of the delta hedge worsens as we rebalance less frequently.

For shorter rebalancing frequencies, the Serenity delta outperforms the Black-Scholes model. We find that the BTC volatility smile introduces complexities which better capture the changes in implied volatility and the option's sensitivity to price movements over shorter rebalancing periods. However, we find that for larger rebalancing frequencies, like the 15-day and 30-day case, the Black-Scholes delta outperforms the Serenity delta.

6 Conclusions

Based on discussions above, the constant volatility assumption of the Black-Scholes model is violated in practice. Hence, we find practical relevance in using deltas that control not only for the option price sensitivity to BTC prices but also the indirect impact from a change in implied volatility. Empirical tests using Serenity's in-house volatility surfaces show that the delta hedging performance of the Black-Scholes model for frequently rehedged strategies can be improved by using our Smile-Adjusted Delta.

Overall, this paper undertook an empirical analysis of delta hedging strategies for Bitcoin options using Serenity's comprehensive derivatives analytics and volatility surfaces, considering the unique characteristics of the digital asset market. By comparing two prominent delta hedging strategies – the classic Black-Scholes delta and Serenity's smile-adjusted delta – the study shed light on the intricate dynamics of risk management in the digital assets space.

Through an in-depth empirical analysis of BTC options, we provided insights into the behavior of implied volatility, capturing its stochastic nature and its sensitivity to market events. The volatility smile, with its distinct patterns during various market events, reflected traders' and investors' responses to tail risks and market sentiment shifts. The comparison of the two delta hedging strategies revealed the superiority of Serenity's smile-adjusted delta in terms of accuracy and stability, especially under more frequent rebalancing. The results showcased the importance of accounting for the volatility smile effect, particularly in the digital assets market, where risk dynamics are far from the assumptions of traditional financial models.

7 Authors

Sonakshi Rohra, Quantitative Researcher @ Cloudwall

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[*] Cloudwall and the technology behind its Serenity System were acquired by Talos in April 2024.

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